WATER HAMMER IN HYDRAULIC PIPE LINES

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Being a Theoretical and Experimental Investigation of the Rise or Fall in Pressure in a Pipe Line, caused by the gradual or sudden closing or opening of a Valve; with a Chapter on the Speed Regulation of Hydraulic Turbines

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LONDON

ARCHIBALD CONSTABLE & CO., LTD.

10 ORANGE STREET, LEICESTER SQUARE

1908

D. VAN NOSTRAND COMPANY

NEW YORK

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PREFACE

ALTHOUGH the study of Hydraulics has been very much to the fore during recent years, one particular branch of the subject—viz., that dealing with the problems involved during the acceleration or retardation of a water column, would seem to have been almost entirely neglected.

In fact, except for an experimental investigation by Joukowsky, of which we have no English translation, and which only deals with sudden stoppage of flow, we have no available literature dealing with the matter.

In the majority of text-books on Hydraulics the treatment which the question receives is not only of the scantiest but is as inaccurate as scanty, and as the whole subject is most interesting and is certainly not without practical importance, perhaps it is unnecessary to apologise for the following account of a theoretical and experimental investigation carried out by the author.

The experimental portion of the work was done in the Engineering Laboratories of the Manchester University, and the thanks of the author are due to the University authorities for their ready acquiescence in this publication of the work, and to Mr. J. Hall of the Engineering Staff for much help in the preparation of the apparatus and in carrying out the experiments.

A. H. GIBSON.

MANCHESTER, *September* 1908.

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CHAPTER I

Introduction—General Theory of Gradual Closure—Description of Experimental Apparatus—Experimental Results—Gradual opening of a Valve.

1. Introduction.

It is a matter of general knowledge, that any retardation or acceleration of a water column, such as may be caused by the closing or opening of a valve at the outlet from a pipe line, is accompanied by a rise or fall in the pressure behind the valve.

In certain cases this change in pressure is transmitted equally to every portion of the pipe line.

When such a change takes the form of a sudden rise in pressure, this is known as Water Hammer. Its magnitude may be large with even comparatively low velocities of flow, and it is of importance, not only in so far as it affects the safety of a pipe line, but also in its effect on the ease of regulation of any prime mover supplied by the latter. In this latter respect the fall in pressure caused by opening the regulating sluice is of even greater importance than a corresponding rise in pressure, since the latter may be readily guarded against by the provision of an adequate relief valve or pressure regulator.

Although the rise in pressure caused by the sudden closing of a valve has been investigated by more than one experimenter, the pressure change due to a gradual closure, or to a gradual opening, has not, to the author's knowledge, been

the subject of previous investigation. Further, those formulæ which have, in one or two instances, been educed as giving such pressures, may be shown to be based on assumptions which are fundamentally unsound, and when tested show results which are in general widely at variance with those obtained experimentally.

It was these facts which led to the following theoretical and experimental investigation of the subject.

2. Gradual Closure of a Valve—General Theory.

If a rigid body of weight W lbs. has its velocity in any given direction changed by an amount δv ft. per sec., in a time δt seconds, its acceleration is $\delta v/\delta t$, and the change of momentum per second in this direction is $\frac{W}{g}\cdot\frac{\delta v}{\delta t}$ in gravitational units.

To produce this acceleration or change in momentum, a force F must be applied to the body in the direction in which this change takes place, its magnitude being given by the formula :—

$$F=\frac{W}{g}\cdot\frac{\delta v}{\delta t}\text{ lbs.}$$

As an example of this, if water be flowing along a rigid pipe whose cross-sectional area at any particular point is a sq. ft., and if $\frac{dv}{dt}$ be its acceleration, the force necessary to accelerate a short element of the column at this point, of length δx, will equal

$$\frac{w}{g}\,.\,a\,.\,\delta x\,.\,\frac{dv}{dt}\text{ lbs.,}$$

all dimensions being in feet and w being the weight (62·4 lbs.) of 1 cubic foot of water.

This force can only be produced by a difference of pressure

at the two ends of the element, and, if $\delta p'$ be this pressure difference in lbs. per sq. ft., we have

$$\delta p' = \frac{w}{g} \cdot \delta x . \frac{\delta v}{\delta t}.$$

In a pipe of length l ft. the difference of pressure at the two ends, due to acceleration of the column $= \Sigma \delta p' = p'$, so that

$$p' = \frac{w}{g} \int_0^l \frac{dv}{dt} . dx \text{ lbs. per sq. ft.} \quad \ldots \ldots \quad (1)$$

If the pipe be of uniform area, and if water be assumed to be an incompressible fluid so that, no matter what changes of pressure may occur throughout its mass, each particle has the same velocity at any given instant, we get, at this instant

$$p' = \frac{wl}{g} \cdot \frac{dv}{dt} \text{ lbs. per sq. ft.}$$

Where this measures the change in pressure at the outlet valve, it will be positive when the motion is being retarded and negative when being accelerated, so that the equation must be written

$$p' = -\frac{wl}{g} \cdot \frac{dv}{dt} \text{ lbs. per sq. ft.} \quad \ldots \ldots \quad (1')$$

This pressure difference at the two ends of the pipe is superposed on that due to steady flow with the velocity obtaining at the given instant. Thus if the loss of pressure from entrance to exit, due to steady flow with velocity v ft. per sec., is $\frac{wv^2}{2g}\left(1+\frac{fl}{m}\right)$ lbs. per sq. ft.,* and if v and $\frac{dv}{dt}$ are re-

* Here $\frac{v^2}{2g}$ measures the loss of pressure head in feet, due to change from pressure to kinetic energy, while $\frac{flv^2}{2gm}$ measures the loss of head due to pipe friction. In this expression m is the hydraulic mean depth of the pipe and in a circular pipe has the value (diam. $\div 4$), while f is a numerical co-efficient whose value depends on the physical conditions of the pipe walls and to a less extent on the diameter of the pipe and on the velocity of flow. See. *Hydraulics*, Gibson, pp. 202-204.

spectively the velocity and the acceleration at a given instant, the pressure at the valve will be less than the statical pressure under conditions of no flow, by an amount

$$\frac{w}{g}\left\{l\frac{dv}{dt}+\frac{v^2}{2}\left(1+\frac{fl}{m}\right)\right\} \text{ lbs. per sq. ft.}$$

Where $\frac{dv}{dt}$ is constant the state of affairs is shown graphically in Fig. 1. Here $AC=p_s$ represents the statical

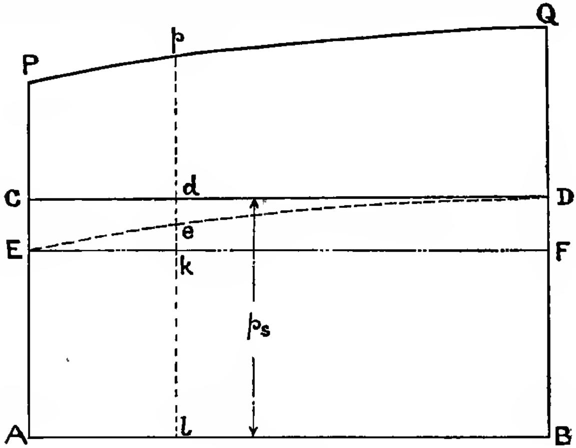

FIG. 1.

pressure at the valve when the latter is closed; AE is the pressure with the valve fully open and under steady flow with velocity v_1 so that $CE=\frac{wv_1^2}{2g}\left\{1+\frac{fl}{m}\right\}$ lbs. per sq. ft.; while the intercept ed, between the curve ED and the horizontal CD, represents $\frac{wv^2}{2g}\left\{1+\frac{fl}{m}\right\}$ where v is the velocity at d. The curve PpQ is parallel to EeD, $EP=ep=DQ$ being equal to $-\frac{wl}{g}\cdot\frac{dv}{dt}$. The rise above statical pressure at the

valve at the instant d is thus given by the ordinate pd, $\left[pd=-\frac{w}{g}\left\{l\frac{dv}{dt}+\frac{v^2}{2g}\left(1+\frac{fl}{m}\right)\right\}\right]$, while the actual pressure is given by the ordinate pl.

Under these conditions the pressure in the pipe evidently has its maximum value BQ, at the instant the valve reaches its seat, this, at the valve, being given by

$$p=p_s-\frac{wl}{g}\left(\frac{dv}{dt}\right) \text{ lbs. per sq. ft.}$$

and thus being independent of frictional losses in the pipe.

Whether the pipe line is uniform in section or not, and whether the acceleration is uniform or variable, the state of affairs is represented by the general equation,

$$\frac{d}{dx}\left\{\frac{p}{w}+\frac{v^2}{2g}+z\right\}=-\left\{\frac{1}{g}\frac{dv}{dt}+f\frac{v^2}{2gm}\right\} \quad \ldots\ldots \quad (2)^*$$

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* As is proved in any treatise on Hydraulics, during the steady non-sinuous flow of any non-viscous incompressible fluid the energy per lb. remains constant, as expressed by the relationship $\frac{p}{w}+\frac{v^2}{2g}+z=$ constant. This theorem is called after its discoverer, Bernoulli.

In the case of a viscous fluid such as water, however, this is not quite true, since energy is lost in overcoming viscous or frictional resistances. If, as is commonly the case, the work done against such resistances is expressed as $\frac{fv^2}{2gm}$ per lb. per unit length of a pipe through which flow is taking place, the equation becomes

$$\frac{d}{dx}\left(\frac{p}{w}+\frac{v^2}{2g}+z\right)=-\frac{fv^2}{2gm}.$$

If in addition the velocity at any point varies with the time, so that the acceleration is $\frac{dv}{dt}$, the force per lb. necessary to produce this acceleration is $\frac{1}{g}\frac{dv}{dt}$, and the work done by this force while motion takes place through a distance δx is $\frac{1}{g}\cdot\frac{dv}{dt}\cdot\delta x$.

Since this work is done at the expense of the energy in the water, the equation as finally modified for friction and for acceleration becomes

$$\frac{d}{dx}\left\{\frac{p}{w}+\frac{v^2}{2g}+z\right\}=-\left\{\frac{1}{g}\frac{dv}{dt}+\frac{fv^2}{2gm}\right\}.$$

where z is the height of some point in the fluid above datum level in ft., x being its distance from some arbitrary point, measured in the direction of flow ; v is the velocity and $\frac{dv}{dt}$ the rate of change with time of the velocity, at this point.

This, which is Bernoulli's equation modified to suit the accelerated motion of a viscous fluid, expresses the fact that the rate of change of energy (pressure + kinetic + potential) per lb. of water in the direction of flow, is equal to the force necessary to produce the required acceleration, minus the force equivalent to pipe friction.

Integrating both sides of (2) with respect to x we get

$$\frac{p}{w}+\frac{v^2}{2g}+z=-\frac{1}{g}\int_0^x\frac{dv}{dt}.dx-\frac{f}{2gm}\int_0^x v^2dx+c \quad . \quad . \quad . \quad . \quad (3)$$

Whenever the acceleration is a known function of the time or of the distance travelled by a particle, equation (3) may be solved and the pressure at any point obtained.

As an example, consider the uniformly retarded flow through a pipe of uniform cross-sectional area and of length l. Let $\frac{dv}{dt}=-a$, and let the suffixes v and 1 refer to the pipe immediately behind the valve and to the inlet at the top respectively. Let x be measured from the inlet (Fig. 2), and let v_a be the velocity in the pipe line.

Then at the inlet, where $x=0$, we have $p=p_1$, $z=z_1$,

$$\int_0^x v^2.dx=0, \text{ so that } c=\frac{p_1}{w}+\frac{v_a^2}{2g}+z_1 \text{ from (3)}$$

when $x=l$, *i.e.* behind the valve, we have $p=p_v$, $z=z_v$,

$$\int_0^l v^2dx=v_a^2 l,\quad \int_0^l\frac{dv}{dt}dx=-a\int_0^l dx=-al\,;$$

$$\therefore\quad \frac{p_v}{w}+\frac{v_a^2}{2g}+z_v=\frac{a}{g}.l+\frac{p_1}{w}+\frac{v_a^2}{2g}+z_1-\frac{flv_a^2}{2gm} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

But $\frac{p_1}{w}+z_1+\frac{v_a^2}{2g}-z_v$ is the head equivalent of the statical

pressure p_s at the valve with no flow through the pipe, so that we get

$$\frac{p_v}{w}=\frac{a}{g}l+\frac{p_s}{w}-\frac{v_a^2}{2g}\left\{1+\frac{fl}{m}\right\}\text{feet of water,} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

or $$p_v=p_s+\frac{w}{g}\left\{al-\frac{v_a^2}{2}\left(1+\frac{fl}{m}\right)\right\}\text{lbs. per sq. ft.,}$$

the result previously obtained from general considerations. Obviously this expression has its maximum value when $v_a=0$, *i.e.* at the instant the valve reaches its seat.

In order to get uniform retardation of a column by closing a valve at its lower end, the rate of closure of this valve would

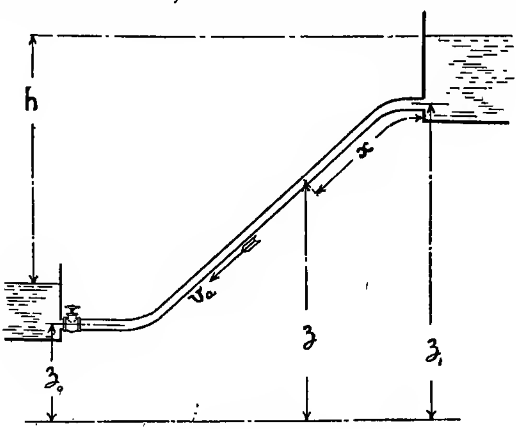

FIG. 2.

however need to be somewhat complicated. If a is the pipe area, and a_0 the effective valve area at any instant (the effective valve area is the actual area multiplied by the coefficient of discharge), and if v_0 is the corresponding velocity of efflux, v_a being the corresponding velocity of pipe-flow, we have $v_a=\frac{a_0v_0}{a}$, so that

$$\frac{dv_a}{dt}=\frac{1}{a}\left\{a_0\frac{dv_0}{dt}+v_0\frac{da_0}{dt}\right\}.$$

If a_1 is the effective valve area when the valve begins to close, or if v_1 is the corresponding velocity of pipe-flow, the value of a_0 after t seconds is given by

$$a_0 = a\left\{\frac{v_1+\alpha t}{\sqrt{2\alpha l+2gh-\frac{fl}{m}(v_1+\alpha t)^2}}\right\}.*$$

In an experiment carried out by Mr. I. P. Church,† on a pipe line 8 inches diameter, and 2395 feet long, fitted with a nozzle 2 inches diameter at its lower end and discharging into the atmosphere, the nozzle was closed in 25 seconds, so as, it is stated, to give uniform retardation in the pipe. The statical head at the valve was 302 feet (131 lbs. per sq. in.), and the pressure at the nozzle during steady flow was 108 lbs. per sq. in. The maximum pressure attained was 143 lbs. per sq. in., so that $p_v - p_s = 12$ lbs. per sq. in.

Here, assuming a co-efficient of velocity of ·985, the velocity of efflux would be $\cdot 985\sqrt{2g\times 108\times 2\cdot 31} = 125$ ft. per sec., so that the velocity in the pipe would be 7·81 ft. per sec.

This makes $-\alpha = \cdot 312$ ft. per sec. per sec., and makes

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* If $p=p_0$, $z=z_0$ on the exit side of the valve, assuming that the energy per lb. remains constant during the passage through the valve, we have $\frac{p_v}{w}+\frac{v_a^2}{2g}+z = \frac{p_0}{w}+\frac{v_0^2}{2g}+z_0$, so that (5) above becomes $\frac{p_0}{w}+\frac{v_0^2}{2g} = \frac{\alpha l}{g}+\frac{p_s}{w}-\frac{v_a^2}{2g}\frac{fl}{m}$. Putting $\frac{p_s-p_0}{w}=h$, where h is the difference of level of the free surfaces on the two sides of the valve, this becomes

$$v_0 = \sqrt{2\alpha l+2gh-\frac{fl}{m}v_a^2}.$$

From this we get $a_0 = \frac{v_a a}{v_0} = \frac{v_a a}{\sqrt{2\alpha l+2gh-\frac{fl}{m}v_a^2}}$, and since $v_a = v_1+\alpha t$, where t is the time since the valve began to close, this gives us both the velocity of efflux and the valve opening at this instant.

† *Journal of Franklin Institute.* April and May 1890.

$$-\frac{w}{g}\cdot al=\frac{62\cdot4\times\cdot312\times2395}{32\cdot2}=1455 \text{ lbs. per sq. ft.}$$
$$=10\cdot1 \text{ lbs. per sq. in.},$$

as compared with the observed value 12 lbs. per sq. in.

Uniform Closing of a Valve.

The most useful case in practice, however, appears to be that in which the valve is closed uniformly, and it is this case to which the present investigation has been more particularly applied.

Let the pipe-line, of uniform area a sq. ft., discharge at its lower end through a valve into a chamber where the pressure is uniformly p_0 lbs. per sq. ft. Let times be measured *backward* from the instant the valve reaches its seat, so that, if the valve be closed uniformly in T seconds, using the same notation as before we have $a_0=a_1\frac{t}{T}$ as giving the valve opening at an instant t secs. *before closure is complete.* Let v_a be the velocity in the pipe line. Equation (3) now becomes

$$\frac{p}{w}+\frac{v^2}{2g}+z=\frac{1}{g}\int_0^x\frac{dv}{dt}\cdot dx-\frac{f}{2gm}\int_0^x v^2dx+c \quad . \quad . \quad . \quad . \quad (6)$$

δt being negative.

Also since, when $x=0$, $\left\{\begin{array}{l} p=p_1,\ z=z_1,\ \int_0^x\frac{dv}{dt}dx=0, \\ v=v_a,\ \int_0^x v^2dx=0, \end{array}\right\}$

$$\therefore\ c=\frac{p_1}{w}+\frac{v_a{}^2}{2g}+z_1.$$

Again, when $x=l$, *i.e.* on the outlet side of the valve, neglecting losses in the valve, we have

$$\left\{\begin{array}{ll} p=p_0,\ z=z_0, & \int_0^l\frac{dv}{dt}\cdot dx=l\frac{dv}{dt} \\ v=v_0,\ \int_0^l v^2dx=v_a{}^2l, & \end{array}\right\};$$

so that $\dfrac{v_0^2-v_a^2}{2g}+\dfrac{p_0-p_1}{w}+z_0-z_1=\dfrac{l}{g}\left\{\dfrac{dv_a}{dt}-\dfrac{fv_a^2}{2m}\right\}$ . . . . . (7)

Writing $\dfrac{p_1}{w}+\dfrac{v_a^2}{2g}+z_1-\dfrac{p_0}{w}-z_0=h$ feet, where h is the difference of statical head on the two sides of the valve with no flow taking place, equation (7) becomes

$$\frac{v_0^2}{2g}-h=\frac{l}{g}\left\{\frac{dv_a}{dt}-\frac{fv_a^2}{2m}\right\} \quad . . . . (8)$$

On substituting for $\dfrac{dv_a}{dt}$, its value $\dfrac{1}{a}\left\{a_0\dfrac{dv_0}{dt}+v_0\dfrac{da_0}{dt}\right\}$, this becomes $a_0\dfrac{dv_0}{dt}+v_0\dfrac{da_0}{dt}=\dfrac{a}{l}\left\{\dfrac{v_0^2}{2}\left(1+\dfrac{a_0^2}{a^2}\dfrac{fl}{m}\right)-gh\right\}$ . . . . (9)

and on dividing throughout by the co-efficient of v_0^2, we get

$$ka_0\frac{dv_0}{dt}=v_0^2-bv_0-c,$$

or $$\int\frac{dv_0}{v_0^2-bv_0-c}+D=\frac{1}{k}\int\frac{dt}{a_0}=\frac{T}{ka_1}\int\frac{dt}{t} \quad . . . . . (10)$$

where $b=\dfrac{\dfrac{2l}{a}\cdot\dfrac{da_0}{dt}}{1+\dfrac{fl}{m}\left(\dfrac{a_0}{a}\right)^2}$; $c=\dfrac{2gh}{1+\dfrac{fl}{m}\left(\dfrac{a_0}{a}\right)^2}$; $k=\dfrac{2l}{a\left\{1+\dfrac{fl}{m}\left(\dfrac{a_0}{a}\right)^2\right\}}$.

If a_0/a is small, so that the term $\dfrac{fl}{m}\left(\dfrac{a_0}{a}\right)^2$ is small in comparison with unity, and may therefore be neglected, b, c, and k, become constants having the values

$$b=\frac{2l}{a}\cdot\frac{da_0}{dt};\ c=2gh;\ k=\frac{2l}{a}.$$

This will always be the case as the valve gets close to its seat, and when in consequence the hammer effect is most noticeable. In such a case both sides of (10) become integrable. Writing this in the form

$$-\int\frac{dv_0}{\left(\sqrt{c+\frac{b^2}{4}}\right)^2-\left(v_0-\frac{b}{2}\right)^2}+D=\frac{T}{ka_1}\log_e t \quad . . (11)$$

and determining D from the consideration that when $t=T$, v_0 has a known value $\bar{v}_0$, we have

$$\frac{1}{2\sqrt{c+\frac{b^2}{4}}}\log_e\left\{\frac{\sqrt{c+\frac{b^2}{4}}-\frac{b}{2}+v_0}{\sqrt{c+\frac{b^2}{4}}+\frac{b}{2}-v_0}\cdot\frac{\sqrt{c+\frac{b^2}{4}}+\frac{b}{2}-\bar{v}_0}{\sqrt{c+\frac{b^2}{4}}-\frac{b}{2}+\bar{v}_0}\right\}=$$

$$\frac{T}{ka_1}\log_e\frac{T}{t}$$

$$\text{or } \frac{1}{m}\log_e\left\{\frac{r+v_0}{q-v_0}\cdot\frac{q-\bar{v}_0}{r+\bar{v}_0}\right\}=\frac{T}{ka_1}\log_e\frac{T}{t} \quad . \quad . \quad . \quad . \quad (12)$$

$$\therefore\ v_0=\frac{\frac{r+\bar{v}_0}{q-\bar{v}_0}\cdot q\cdot\left(\frac{T}{t}\right)^{\frac{mT}{ka_1}}-r}{1+\frac{r+\bar{v}_0}{q-\bar{v}_0}\left(\frac{T}{t}\right)^{\frac{mT}{ka_1}}} \text{ ft. per sec.} \quad . \quad . \quad . \quad . \quad (13)$$

where $r=\sqrt{c+\frac{b^2}{4}}-\frac{b}{2}$; $q=\sqrt{c+\frac{b^2}{4}}+\frac{b}{2}$; $m=2\sqrt{c+\frac{b^2}{4}}$

This gives the velocity of efflux, from which the velocity v_a at any instant within the range of valve opening over which $\frac{fl}{m}\left(\frac{a_0}{a}\right)^2$ is negligibly small, may be readily obtained.*

Writing p_v for the difference of pressure on the two sides of the valve at any instant during closure, we get, as in equation (5),

$$p_v=p_s+\frac{w}{g}\left\{\frac{dv_a}{dt}\cdot l-\frac{v_a^2}{2}\left(1+\frac{fl}{m}\right)\right\} \text{ lbs. per sq. ft.}$$

and, on substituting for $\frac{dv_a}{dt}$ from (8) this becomes

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* Where $\frac{fl}{m}\left(\frac{a_0}{a}\right)^2$ is not negligible the treatment follows the lines outlined on page 28. This case is, however, not of great practical importance, since the rise in pressure before the valve gets near to its seat, and hence before the state of affairs hypothecated obtains, is usually very small.

$$p_v = p_s + \frac{w}{g}\left\{\frac{v_0^2}{2} - \frac{v_a^2}{2} - gh\right\} \text{ lbs. per sq. ft., } \quad . \; . \; . \quad (14)$$

Evidently this has its maximum value when v_0 is a maximum and therefore, from (13), when t vanishes, *i.e.* at the instant the valve reaches its seat. At this instant v_0 attains the limiting value q, while v_a becomes zero, so that we get

$$(p_v)_{max} = p_s + \frac{w}{g}\left\{\frac{q^2}{2} - gh\right\} \text{ lbs. per sq. ft.}$$

But $\frac{q^2}{2} - gh = \frac{c}{2} + \frac{b^2}{4} + \frac{b}{2}\sqrt{c + \frac{b^2}{4}} - gh$

$$= \left(\frac{l}{a}\cdot\frac{da_0}{dt}\right)^2 + \frac{l}{a}\cdot\frac{da_0}{dt}\cdot\sqrt{2gh + \left(\frac{l}{a}\cdot\frac{da_0}{dt}\right)^2}$$

It follows that the rise in pressure behind the valve at the instant when closure is complete, above that obtaining with no flow through the pipe, is given by

$$p' = \frac{w}{g}\left[\left(\frac{l}{a}\cdot\frac{a_1}{T}\right)^2 + \frac{l}{a}\cdot\frac{a_1}{T}\sqrt{2gh + \left(\frac{l}{a}\cdot\frac{a_1}{T}\right)^2}\,\right] \text{ lbs. per sq. ft.} \quad (15)$$

where a_1 is the maximum effective valve opening in sq. ft., and T is the time in seconds, taken to close the valve uniformly.

Pipe of varying cross-sectional areas.

If the pipe line, instead of being uniform in section, consists of a length l_x of sectional area a_x, a length l_y of area a_y, and so on, we have, at any instant:—

Velocity in section $x = v_x = \frac{v_0 a_0}{a_x}$

$\therefore$ Acceleration ,, ,, $x = \frac{dv_x}{dt} = \frac{1}{a_x}\left\{v_0\frac{da_0}{dt} + a_0\frac{dv_0}{dt}\right\}$

Similarly ,, ,, ,, $y = \frac{dv_y}{dt} = \frac{1}{a_y}\left\{v_0\frac{da_0}{dt} + a_0\frac{dv_0}{dt}\right\}$

It may then be readily shown that the maximum rise in

pressure behind the valve, at the instant of closure, is given by:—

$$p'=\frac{w}{g}\left[\left(\frac{a_1}{T}\right)^2\left(\Sigma\frac{l}{a}\right)^2+\frac{a_1}{T}\sqrt{2gh+\left(\frac{a_1}{T}\right)^2\left(\Sigma\frac{l}{a}\right)^2}\cdot\Sigma\frac{l}{a}\right]\text{lbs. per sq. ft.} \ldots (16)$$

So far the effects of the elasticity of the pipe line and of the compressibility of the water column have been neglected. These factors, however, tend to reduce the maximum pressure attained, and the extent to which their effect is likely to modify the results obtained above, may be seen as follows:—

Bernoulli's Equation as modified for the elasticity of the water column.

In any elastic fluid, where w is not constant but depends on the pressure, on neglecting friction, equation (2), p. 5, becomes

$$\frac{d}{dx}\left(\int\frac{dp}{w}+\frac{v^2}{2g}+z\right)=-\frac{1}{g}\,\frac{dv}{dt}\,. \quad (2')$$

Now in water, if V be the volume of unit mass at atmospheric pressure ($p=0$), we have $-\frac{\delta V}{V}=\frac{\delta p}{K}$. $K=\frac{\delta p}{\delta V}\frac{1}{V}$

where K is the modulus of compressibility of the water and has a mean value of about 43,200,000 lbs. per sq. foot.

From this we have $\frac{V-\delta V}{V}=1+\frac{\delta p}{K}$, or, if V' is the volume corresponding to a pressure p lbs. per sq. foot above atmospheric, $\frac{V'}{V}=1-\frac{p}{K}$. V = vol at atmos pressure

If w' and w are the corresponding weights per cubic foot this gives us $\frac{w}{w'}=1-\frac{p}{K}$. w = #/cu ft at atmos pressure

From this we get $\int\frac{dp}{w'}=\frac{1}{w}\int\left(1-\frac{p}{K}\right)dp=\frac{p}{w}-\frac{p^2}{2Kw}$.

So that equation (2′) becomes

$$\frac{p}{w}+\frac{v^2}{2g}+z-\frac{p^2}{2Kw}=-\frac{1}{g}\int\frac{dv}{dt}+c. \quad (3')$$

the term $\frac{p^2}{2Kw}$* representing the resilience, or strain energy of the liquid, per lb.

Where the liquid is being compressed in an elastic pipe this requires further modification because of the fact that part of the energy of the water is expended in stretching the pipe.

This modifies the apparent value of K, and, if the effect of the pipe supports be neglected, may be taken into account as follows :—

Effect of elasticity of pipe line.

Suppose the pipe to be of radius r ft., and of comparatively small thickness t ft., and let the material of which it is composed have a modulus of elasticity E lbs. per sq. ft., and a Poisson's ratio $\frac{1}{\sigma}$.

Then if at any section of the pipe the increase in pressure due to retardation is p' lbs. per sq. ft., the increase in the

* If the rise in pressure is p' lbs. per sq. ft., the increase in resilience per cubic foot is $\frac{p'^2}{2K}$, and therefore per lb. is $\frac{p'^2}{2Kw}$. For consider a cubical block whose sides are of unit length, subjected to a pressure which increases steadily from zero to p'. The change in volume of the block is $p' \div K$, and therefore the change in length of each side is $p' \div 3K$ (very approx.). But during the period of compression the mean value of the pressure is half the final pressure p, so that the work done by each of the three opposite pairs of pressures is $\frac{p'}{2}\times\frac{p'}{3K}=\frac{p'^2}{6K}$.

$$\therefore \left.\begin{array}{l}\text{Total work done on block,}\\ \textit{i.e.}\text{ resilience stored in block}\end{array}\right\}=\frac{p'^2}{2K}.$$

circumferential stress in the pipe walls is $\frac{p'r}{t}$ and in the longitudinal stress is $\frac{p'r}{2t}$ lbs. per sq. ft.

If, then, δx is the change in length of an element of the pipe at this point, whose original length was x, we have

$$\frac{\delta x}{x}=\frac{p'r}{2tE}-\frac{p'r}{\sigma tE},$$

$$\therefore\quad \delta x=x\frac{p'r}{2tE}\left\{1-\frac{2}{\sigma}\right\},$$

$$\text{Also}\quad \frac{\delta r}{r}=\frac{p'r}{tE}-\frac{p'r}{2\sigma tE},$$

$$\therefore\quad \delta r=r\frac{p'r}{tE}\left\{1-\frac{1}{2\sigma}\right\}.$$

The change in the volume of this element is therefore given by

$$\pi\{(r+\delta r)^2(x+\delta x)-r^2x\},$$

$$=\pi\{2rx\delta r+r^2\delta x\}\ \left\{\begin{array}{l}\text{neglecting small quantities}\\ \text{of the second order.}\end{array}\right.$$

$$=\pi r^2x\left\{\frac{2p'r}{tE}\left(1-\frac{1}{2\sigma}\right)+\frac{p'r}{2tE}\left(1-\frac{2}{\sigma}\right)\right\}$$

$$=\pi r^2x\times\frac{p'r}{2tE}\left\{5-\frac{4}{\sigma}\right\}$$

$$\therefore\quad \frac{\delta V}{V}=\frac{p'r}{2tE}\left\{5-\frac{4}{\sigma}\right\}.$$

But the actual new volume of the liquid $=\pi r^2x\left(1-\frac{p'}{K}\right)$

while its apparent new volume $=\pi r^2x\left\{1-\frac{p'}{K}-\frac{p'r}{2tE}\left(5-\frac{4}{\sigma}\right)\right\}$,

so that the effective value of K, which will be denoted by K', is given by the relation

$$\frac{1}{K'}=\frac{1}{K}+\frac{r}{2tE}\left(5-\frac{4}{\sigma}\right).$$

For iron pipes σ may be taken as 3·6 approximately.

From equation (3′) it appears that where flow is taking place so that Bernoulli's equation holds, the ratio of the resilient energy per lb. to the pressure energy per lb. being $\frac{p}{2K'}$, this will in all cases be so small as to be negligible.

In the author's experiments the value of K' was found to be 251,000 lbs. per sq. inch,* so that where p is as great as 100 lbs. per sq. inch this ratio is only ·0002.

A further effect of the elasticity of the column is, however, more important—and, in fact, is all-important where the time of closing is very short.

This is due to the fact that, because of the elasticity the effect of any change in the pressure, velocity, or acceleration at the valve, will not be felt at a distance x along the pipe, until an interval of time $x \div V_p$ seconds has elapsed. Here V_p is the velocity of propagation of pressure waves along the water column and has a value generally ranging from 4200 to 4600 ft. per second.

If at an instant t seconds before it reaches its seat the retardation at the valve is represented by $\phi(t)$, the retardation at a distance x will therefore be the same as at the valve at an instant $x \div V_p$ seconds sooner, and will be

* As will be seen later, the velocity of propagation of pressure waves through water is given by $V_p = \sqrt{\frac{K'g}{w}}$ ft. per second, where K' is in lbs. per sq. foot. In these experiments this velocity was measured by closing the outlet valve suddenly and timing the oscillations of the pencil lever of a Crosby indicator mounted on the pipe. In this way it was found that the velocity of propagation was 4310 ft. per second, this making $K' = 251,000$ lbs. per sq. inch. Assuming $K = 300,000$ lbs. per sq. inch, and taking $r = 3{\cdot}3t$ as in this experimental pipe line, this makes $E = 10^7$ lbs. per sq. inch. As these pipes are cast-iron, and have spigot and faucet joints with lead caulking, this would appear a very probable value for E.

represented by $\phi\left(t+\frac{x}{V_p}\right)$. This is true at all times, after an instant $\frac{x}{V_p}$ seconds from the beginning of the closure.

If the time of closing is greater than $l \div V_p$, we have therefore at the instant of closure, on equating Σ (mass$\times$ acceleration) over the whole length of pipe to the force producing this acceleration:— an acceleration

$$p'a=\frac{aw}{g}\int_0^l \phi\left(t_{=0}+\frac{x}{V_p}\right)dx \quad . \ . \ . \ . \ . \ . \ . \ . \ (17)$$

Now $\phi(t_{=0})$ is the retardation at the valve at the instant of closing, so that the second term, which is negative, marks the effect of elasticity in reducing the maximum pressure attained at the valve. This may be evaluated for the case under consideration by substitution for $\frac{dv_a}{dt}$ from (8) and by substitution in this expression for $v_0{}^2$ in terms of t from (13).* The expression, however, is too cumbrous for any useful purpose, and simply serves to show that this effect is proportionately greater as the length of pipe increases and as the time of closure diminishes. It is usually of importance for values of $T<4l \div V_p$, but for greater values of T may in general be neglected without serious error.

In such cases formula (15) requires no correction, and the first of the present series of experiments was carried out with a view of testing its accuracy.

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* If the retardation at the valve is uniform, $\phi\left(t+\frac{x}{V_p}\right)=\phi(t)=\frac{da_v}{dt}=a$, at all times after $x \div V_p$ seconds from the beginning of the closure. If the retardation increases uniformly $\phi\left(t+\frac{x}{V_p}\right)=\left(\frac{dv_a}{dt}\right)_{t=0}+\frac{d^2v_a}{dt^2}\cdot\frac{x}{V_p}$, and we get $p'=\frac{wl}{g}\left\{\left(\frac{dv_a}{dt}\right)_{t=0}+\frac{d^2v_a}{dt^2}\cdot\frac{l}{2V_p}\right\}$, the second term being negative.

3. Description of Apparatus.

The experimental portion of the work was made possible by the provision, at the Whitworth Engineering Laboratories of the Manchester University, of a Cast-Iron Pressure Main, 3·75 ins. in diameter and approximately 560 ft. long, this

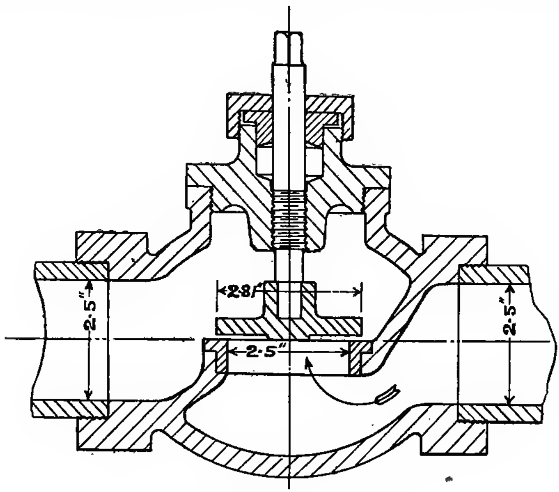

Fig. 3.

bringing water for experimental purposes into the laboratory from an elevated tank 107 ft. above the laboratory floor. The main consists, in chief, of a vertical leg about 110 ft. long and a horizontal leg 450 ft. long, and has eight right-angled bends in the course of its length. The horizontal portion of the pipe is buried, except for the 80 ft. nearest to the outlet, the vertical portion being clamped, at intervals, to a wall, but otherwise being free. The joints throughout are of the spigot and faucet type, with lead caulking, and the thickness of the metal is approximately $\frac{5}{8}$ in.

The main is blanked off at its lower end, and is tapped at a point 18 ft. from this end by a wrought-iron pipe $2\frac{1}{2}$ ins. diameter and 8 ft. long, which carries an outlet valve, $2\frac{1}{2}$ ins. in diameter, and of the design shown in Fig. 3. This, which was used in the experiments, has a brass disc valve, bedding metal-to-metal on the flat brass seat. The pressure of the water tends to keep the valve off its seat.

After passing this valve the water rises through a short vertical leg, passes through a second valve, shown at *B* in fig. 14, this being kept wide open during the experiments, and is led into a tank where its motion is steadied by baffles. It is then allowed to escape over a rectangular notch 4 ins. wide into a lower calibrated tank. The upper tank is provided with a float and graduated scale, from which the quantity per minute passing the notch can be directly ascertained.

This scale, which is in general use for laboratory purposes, is checked at frequent intervals, and may be taken as giving results which are correct within 1 %.

To measure pressures, a Crosby Steam Engine Indicator A, fig. 4, is mounted on the outlet pipe at a point about 5 ft. from the large pipe. The drum of this indicator derives its motion from a cord coiled around the spindle *R*, which carries a pulley and is driven by a leather band from a light pulley mounted on the valve spindle *S*. The spindle *R* is geared to a light cylindrical drum *B*, $5\frac{1}{2}$ ins. diameter, which carries a smoked paper. As the valve spindle rotates, a wave diagram is traced out on this by a fine pointer carried by the tuning fork *C*, which makes 100 complete vibrations per second. Where the time of valve closing exceeds about 5 secs., this fork is replaced by a horizontal pendulum making eight beats per second.

By counting the number of waves corresponding to the travel of the valve, the time of closure can be estimated to

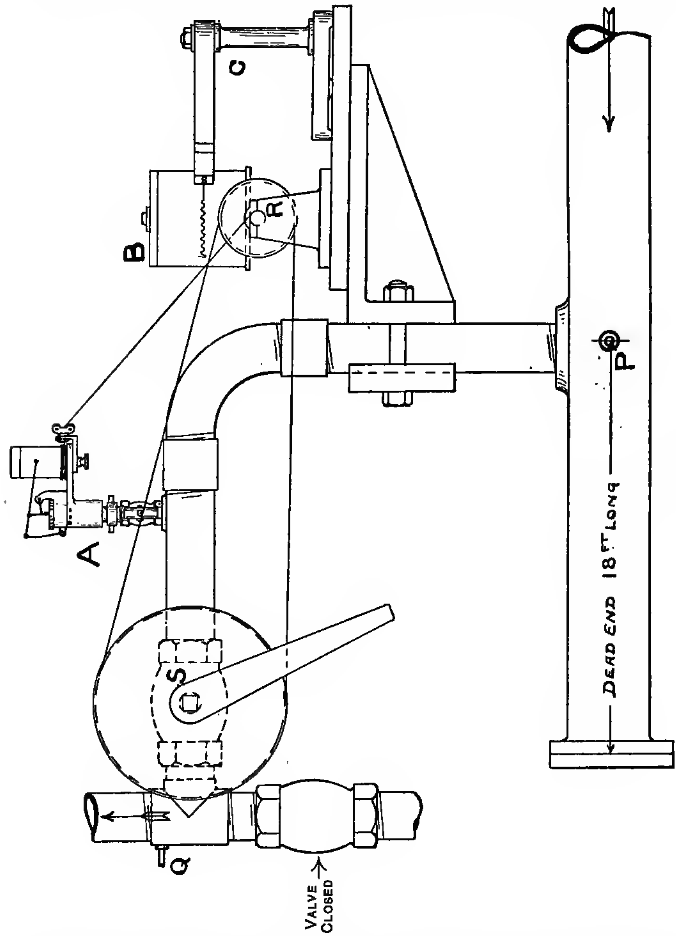

FIG. 4.

the nearest ·005 second in the shorter, and to the nearest ·05 second in the longer experiments, while by counting the

number of waves per unit length of the paper, the rate of closing at any instant can be estimated with considerable accuracy.

In each experiment, the valve spindle was rotated by hand by means of a lever 2 feet long, this being done as steadily and uniformly as possible.

After some practice, it became possible to obtain wave diagrams in this way which showed that the valve motion was sensibly uniform over the whole range of valve opening.

As a check on the accuracy of the indicator, and to determine accurately the resistance to steady flow along the pipe, two mercury gauges are fitted side by side, one coupled to a ½ in. lead pipe leading directly to the elevated supply tank, and thus measuring the statical head, and the second coupled to the foot of the main near the outlet at P.

A third mercury gauge at Q enables the pressure—usually small—at the outlet side of the valve, to be determined.

During all experiments the gauge at P was kept closed.

The pitch of the thread on the screwed valve spindle is ·125 inch, while measurements showed that the discharge area for one turn of the spindle was ·00680 sq. ft. The mean of a number of consistent experiments with different heads and valve openings gave a mean value of ·932 for the co-efficient of discharge, thus giving an effective discharge area of ·00634 sq. ft. for one turn of the spindle. As the area of the main is approximately ·0767 sq. ft., this gives a ratio of pipe area : effective valve area $=12\cdot1$ for one turn of the spindle, and this value has been used in all calculations.

As an examination of equation (16), p. 13, will show, the

effect of the short 5-foot length of 2½ in. pipe at the outlet is practically the same as that of an additional 8-foot length of 3·75 in. pipe, so that in calculating the hammer pressures, formula (15)

$$p'=\frac{w}{g}\left[\left(\frac{l}{a}\cdot\frac{a_1}{T}\right)^2+\frac{l}{a}\cdot\frac{a_1}{T}\sqrt{2gh+\left(\frac{l}{a}\cdot\frac{a_1}{T}\right)^2}\right] \text{ lbs. per sq. ft.}$$

has been used, l being taken as 550 feet.

Although the central portion of the pipe line, being in position, did not admit of very accurate measurement, this is probably within 2 per cent. of the correct value.

4. Experimental Results.

These are tabulated as a whole in the Appendix (Table I.) The following, however, gives a brief *resumé* of the experiments:

Series A. Valve open ·62 of a complete turn; $\frac{a}{a_1}=19{\cdot}5$.

$\frac{flv^2{}_a}{2gm}=28{\cdot}4$ ft. $=12{\cdot}3$ lbs. per sq. inch.

$\bar{v}_0=3{\cdot}03$ ft. per sec.

$h=104{\cdot}6$ ft.

Experimental results plotted at A (Fig. 5). The dotted curve AA' shows the calculated results.

Series B. Valve open ·349 of a complete turn; $\frac{a}{a_1}=34{\cdot}7$.

$\frac{flv_a{}^2}{2gm}=6{\cdot}2$ lbs. per sq. in.

$\bar{v}_0=2{\cdot}155$ ft. per sec.

$h=104{\cdot}6$ ft.

Experimental results plotted at B (Fig. 5). The dotted curve BB' shows results as obtained by calculation.

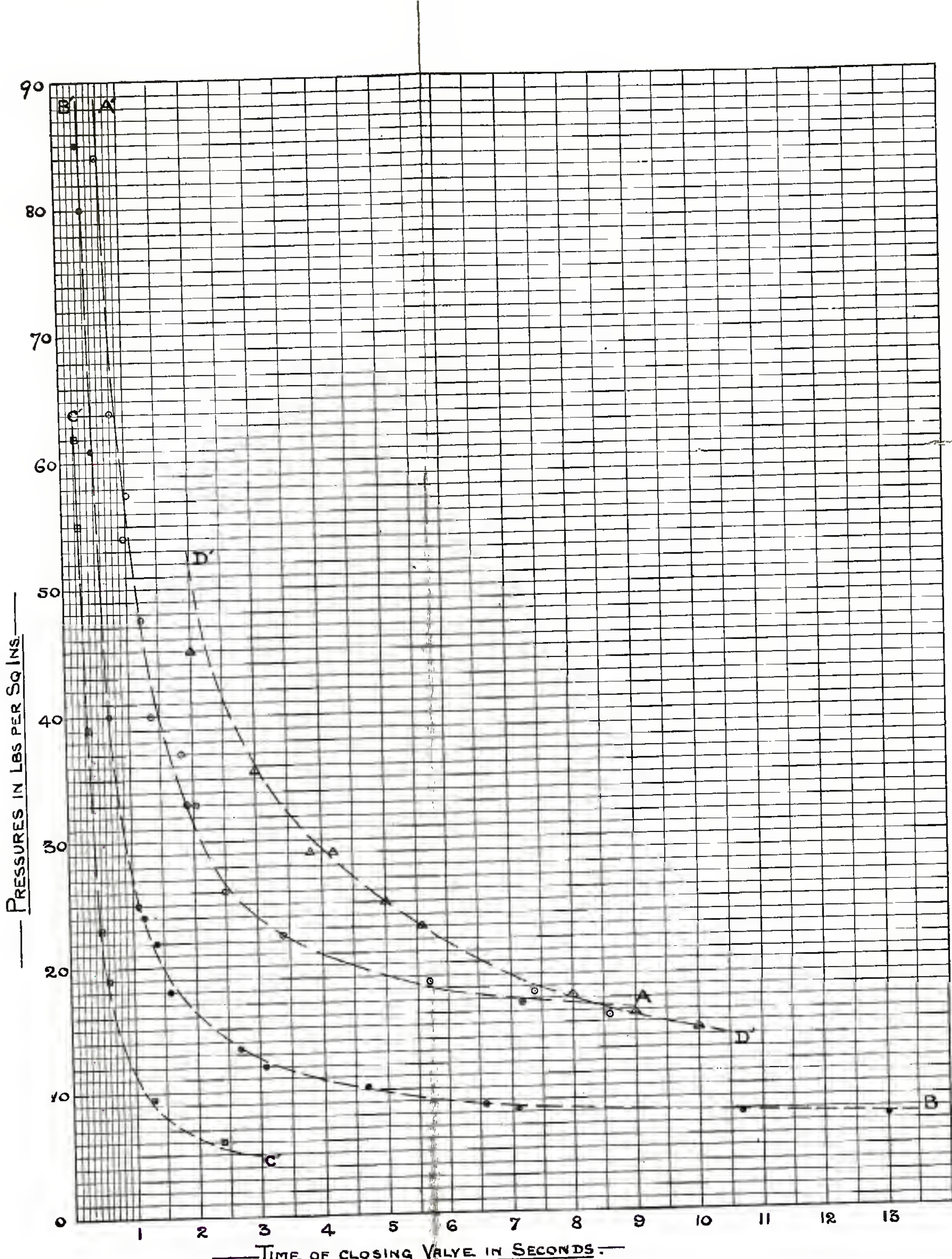

Fig. 5.

Pressures attained behind valve, in excess of those obtaining with steady flow with valve open.

Series C. Valve open ·167 of a complete turn; $\frac{a}{a_1}=72 \cdot 5$.

$\frac{flv_a^2}{2gm}=1 \cdot 6$ lbs. per sq. in.

$\bar{v}_0=1 \cdot 095$ ft. per sec.

$h=105 \cdot 5$ ft.

Experimental results plotted at C (Fig. 5). The dotted curve $C'C'$ shows results as obtained by calculation.

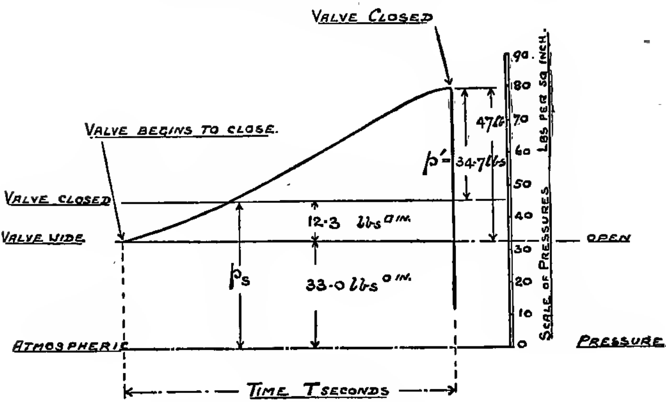

FIG. 6.

Time of closing, 1·25 seconds. Maximum increase over pressure obtaining with steady flow=47 lbs. per sq. in.

Series D. Valve open 1·41 complete turns; $\frac{a}{a_1}=8 \cdot 6$;

$\frac{flv^2}{2gm}=18 \cdot 0$ ft. $=7 \cdot 8$ lbs. per sq. in.

$\bar{v}_0=2 \cdot 405$ ft. per sec.

$h=104$ ft.

Experimental results plotted at D (Fig. 5). Dotted line $D'D'$ shows results as obtained by calculation.

Fig. 7.

Series I. Time of closing 1·25 seconds. Maximum increase above pressure obtaining with steady flow = 47 lbs. per sq. in.

From these results, which cover a range of values of $\bar{v}_0$, *from* 1·095 *to* 3·03 *feet per second, a range of values of the ratio* $a : a_1$ *from* 8·6 *to* 78·6, *and have times of closing varying from* ·22 *to* 13 *seconds, ample verification of the validity of formula* (15) *is obtained.*

Fig. 6 shows a typical indicator diagram, and Fig. 7 the corresponding wave diagram as obtained from one of the experiments of Series I.

When the valve has reached its seat the pressure falls below that corresponding to the statical head, then rises again, a wave of pressure being reflected backwards and forwards along the pipe. This portion of the phenomenon, due solely to the elasticity of the water column, will be explained later. If the indicator drum be rotated uniformly and independently of the valve spindle, a diagram similar to Fig. 8 is obtained.

5. Gradual opening of Valve.

If the valve be gradually opened, so that both valve area and velocity of pipe flow increase with time, equation (6) becomes

$$\frac{p}{w}+\frac{v^2}{2g}+z=-\frac{1}{g}\int_0^x\frac{dv}{dt}dx-\frac{f}{2gm}\int_0^x v^2dx+C \,.\,.\, (6')$$

If the valve is opening uniformly, on proceeding as before (9) becomes

$$a_0\frac{dv_0}{dt}+v_0\frac{da_0}{dt}=-\frac{a}{l}\left[\frac{v_0^2}{2}\left\{1+\left(\frac{a_0}{a}\right)^2\frac{fl}{m}\right\}-gh\right] (9')$$

and on making the assumption that $\left(\frac{a_0}{a}\right)^2\frac{fl}{m}$ is small, this becomes $\int\frac{dv_0}{v_0^2+bv_0-c}+D=-\frac{1}{k}\int\frac{dt}{a_0}$.

When $t=0$, let $v_0=\bar{v}_0$, the flow being steady, and let $a_0=a_1$. Then $a_0=a_1+t\frac{da_0}{dt}=a_1+Kt$, where $K=\frac{da_0}{dt}$.

$$\therefore \left\{\frac{v_0-r}{v_0+q}\cdot\frac{\bar{v}_0+q}{\bar{v}_0-r}\right\}=\left(\frac{a_1}{a_1+Kt}\right)^{\frac{m}{Kk}}$$

from which we get

$$v_0=\frac{\frac{\bar{v}_0-r}{\bar{v}_0+q}\cdot q\left(\frac{a_1}{a_1+Kt}\right)^{\frac{m}{Kk}}+r}{1-\frac{\bar{v}_0-r}{\bar{v}_0+q}\left(\frac{a_1}{a_1+Kt}\right)^{\frac{m}{Kk}}}\text{ ft. per second,}$$

where r, q, m and k have the meanings attached to them on p. 11 viz.,

$$r=\sqrt{c+\frac{b^2}{4}}-\frac{b}{2};\ q=\sqrt{c+\frac{b^2}{4}}+\frac{b}{2};\ m=2\sqrt{c+\frac{b^2}{4}};\ k=\frac{2l}{a}:*$$

This gives the velocity of efflux after an interval of t seconds from the commencement of the motion, in terms of

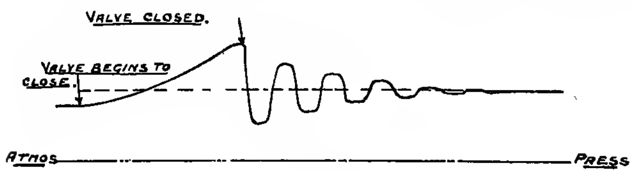

FIG. 8.

the valve opening and velocity of efflux at the latter instant. This only holds so long as the valve is opening. Suppose the valve to be stopped at an instant when its opening is a_0',

* Here $b=\frac{2l}{a}\cdot\frac{da_0}{dt}$ and $c=2gh$ as on page 10.

the velocity of efflux at this instant being calculated to be v_0'. Let $a_0' \div a = n$.

From this time onwards $\frac{dv_a}{dt} = n \cdot \frac{dv_0}{dt}$, and equation (9′) becomes $ln\frac{dv_0}{dt} = -\left[\frac{v_0^2}{2}\left\{1+n^2\frac{fl}{m}\right\} - gh\right]$

So that

$$\int\frac{dv_0}{c'-v_0^2} + D = \frac{1}{k'}\int dt$$

where $c' = \dfrac{2gh}{1+n^2\dfrac{fl}{m}}$; $k' = \dfrac{2ln}{1+n^2\dfrac{fl}{m}}$:

Integrating we get

$$\frac{1}{2\sqrt{c'}}\log\frac{\sqrt{c'}+v_0}{\sqrt{c'}-v_0} + D = \frac{t}{k'}.$$

When $t=0$, *i.e.* immediately the valve comes to rest, $v_0 = v_0'$. Using this to determine D, we finally get

$$v_0 = \frac{\dfrac{\sqrt{c'}+v_0'}{\sqrt{c'}-v_0'}\cdot e^{\frac{2\sqrt{c'}}{k'}\cdot t} - 1}{\dfrac{\sqrt{c'}+v_0'}{\sqrt{c'}-v_0'}\cdot e^{\frac{2\sqrt{c'}}{k'}\cdot t} + 1}\cdot\sqrt{c'} \text{ ft. per sec.}, \quad . \quad . \quad . \quad . \quad (18)$$

as the velocity of efflux after t seconds from the stoppage of the valve.

It will be noted that as t increases, this tends to the limit

$$\sqrt{c'} = \sqrt{\frac{2gh}{1+\dfrac{n^2fl}{m}}}.$$

CHAPTER II

Application of the foregoing work to the theory of turbine regulation —Regulation of a Pelton wheel—of a Pressure turbine—Speed regulation assuming uniform acceleration in the pipe line—Effect of a stand pipe on speed regulation.

1. Application of the foregoing work to the Theory of Turbine Regulation.

In the case of a Pelton wheel installation, where the pipe line is well designed so as to give a maximum velocity of flow of about 5 feet per second, the term $\left(\frac{a_0}{a}\right)^2\frac{fl}{m}$ will in general be negligibly small over the whole range of nozzle opening, and may be treated as being so in equation (9′), p. 24, without sensible error.

For example, consider a Pelton wheel working under a head of 200 ft., and supplied through a pipe-line 2 ft. in diameter and 500 ft. long ($f = \cdot 006$).

Here $\frac{fl}{m} = \cdot 006 \times 500 \times 2 = 6$

and, if $v = 5$, $\frac{flv^2}{2gm} = 2 \cdot 34$ ft.

The velocity of efflux will then be approximately equal to $\cdot 975\sqrt{2g(200 - 2\cdot 34)} = 110$ ft. per sec.

$$\therefore \left(\frac{a_0}{a}\right)_{max} = \tfrac{5}{110} = \tfrac{1}{22}$$

$$\therefore \left\{\left(\frac{a_0}{a}\right)^2\frac{fl}{m}\right\}_{max} = \tfrac{6}{484} = \cdot 0124.$$

In this case the energy lost in the pipe line is little more

than one per cent. of the energy of discharge, while this value will be still further reduced as the valve closes.

Where the valve is replaced by the regulating sluice, or the gates of a pressure turbine, the friction term will still in general be negligibly small where the ratio $l \div h$ is less than about 4.

Where this ratio is large, as where the supply, under a comparatively low head, is brought a considerable distance through a pipe line—a state of affairs which should never be allowed in good practice—this term must, however, be taken into account.

In such a case the difficulty in integrating equations (9) p. 10, or (9′) p. 24, may be overcome by dividing the period during which the gates are in motion, into a series of short intervals during each of which the friction term may be taken as sensibly constant (a_0 having its mean value throughout the interval though varying from interval to interval). If the velocity at any instant be known, the velocity at the beginning or end of the interval containing that instant, and thus the velocity at the beginning or end of the preceding or following interval may thus be obtained. Proceeding in this way the velocity at any instant during the whole period may be obtained. This case is, however, not so important, since in such an installation a stand pipe would invariably be fitted as near to the turbine as practicable, the effect of this being to greatly reduce the acceleration along the supply main.

Neglecting this for the time being, it may be postulated that for successful governing on an increasing load,* the

* By the provision of an adequate relief valve or pressure regulator at the turbine, no difficulty is experienced in governing on a falling load.

inertia of the rotating parts of the turbine must be so great that the energy rendered available during a certain (fixed) reduction in speed (the maximum permissible reduction being strictly limited by the conditions of service), together with the energy entering the turbine during the period of pipe line acceleration, will equal the demand for energy during that period by the driven machine.

Two cases may be considered

(*a*) *Pelton Wheel.*

Here the energy entering the wheel per second, when the jet velocity is v_0 ft. per sec. $=\frac{a_0 v_0^3}{2g}$ ft. lbs.

Suppose the wheel to be running steadily under a load which necessitates an energy supply of N ft. lbs. per second, and suppose the load to be suddenly increased and to demand M ft. lbs. per second.

The original amount of energy entering wheel per second $\Big\} = N = \frac{a_0 \bar{v}_0^3}{2g}$ ft. lbs.

Next let $\frac{da_0}{dt}$, the rate of nozzle opening, be known. If uniform, v_0 is known in terms of a_1 and t, from (18), p. 26, so that, equating $\frac{a_0 v_0^3}{2g}$ to M, we get the time after which the supply is once more equal to the demand.

Let this interval of time $=t_1$.

Then, during this interval, the excess of the energy demanded over that supplied by the water

$$=\int_0^{t_1} \frac{a_0 v_0^3}{2g} dt - Mt_1 \text{ ft. lbs.}$$

and this must be capable of being supplied by the kinetic energy given out by the rotating parts during a fixed reduction of speed.

Thus if ω = angular vel. of rotation when load is thrown on (radians per second)

ω_1 = reduced angular velocity,

I = moment of inertia of wheel

$= \frac{Wr^2}{g}$ where r = its radius of gyration

we have
$$\tfrac{1}{2}I(\omega^2-\omega_1^2)=\int_0^{t_1}\frac{a_0v_0^3}{2g}dt-Mt_1.$$

(b) *Pressure Turbine.*

Exactly the same reasoning and method of solution holds for this class of turbine, except that the energy entering the wheel is now partly in the potential form, and, per second, is given by

$$a_0v_0\left\{\frac{p_0}{w}+\frac{v_0^2}{2g}+z_0\right\}\text{ ft. lbs.}$$

where the suffix 0 refers to the state of affairs at the exit from the guide vanes and at the entrance to the turbine runner.

Now if the datum plane be taken as passing through the turbine $z_0=0$, and if in addition we assume, as is the case with a well-designed turbine running steadily, that $\frac{p_0}{w}$ bears a definite ratio to $\frac{v_0^2}{2g}$, so that $\frac{p_0}{w}=k_1\frac{v_0^2}{2g}$, we have

$$\text{Energy entering wheel per second}=\frac{a_0v_0^3}{2g}\left\{1+k_1\right\}.*$$

* Here k_1 has the following approximate values:—

	k_1	Inlet angle of guide vanes.
Inward radial flow turbine—		
Thomson type . .	1·20	About 11°
Francis type . .	1·2 to ·82	11° to 15°
Mixed flow turbine of the American type . .	·60 to ·30	22° to 28°

N.B.—v_0 is no longer the velocity of efflux due to head h but to a head $h-\frac{p_0}{w}$.

The facts that the turbine is not running quite steadily during this period, and that, owing to the variation in the velocity of inflow, its efficiency is not constant, prevent any accurate calculations being made, however, and when it is remembered how comparatively far our preliminary data may be from being accurate, and that the expressions resulting from this method of treatment are most unwieldy, it appears preferable on all counts, from a practical point of view, to simplify the treatment by assuming that the acceleration in the pipe line is uniform.

On this assumption, which cannot be widely at variance with the conditions actually obtaining in practice, and which is probably more justifiable than that of uniform valve opening in the case of a modern turbine governed through an hydraulic relay, the whole question may be readily examined as follows:—

2. Speed Regulation assuming Uniform Pipe-Line Acceleration.

Let a=area of penstock, at entrance to turbine casing, sq. ft.
,, h=supply head in feet.
,, v_1=velocity of steady flow through pipe line under the least probable load.
,, v_2=velocity of steady flow through pipe line under the greatest probable load.
,, h_s=pressure head at entrance to turbine casing, in feet.

Then when turbine is running steadily under least load, the energy entering casing per second

$$=62{\cdot}4av_1\left\{h-\frac{v_1^2fl}{2gm}\right\}\text{ ft. lbs.}$$

And when running steadily under greatest load this energy per second

$$=62{\cdot}4av_2\left\{h-\frac{v_2^2fl}{2gm}\right\}\text{ ft. lbs.}$$

During the transition period, after the extra load has been thrown on and before the velocity of pipe flow has attained the value v_2 necessary for giving the required supply, the water column is being accelerated, and during this period, if α is the uniform acceleration, and v the velocity, we have, at an instant t seconds after throwing on the load :—

$$\text{Energy entering wheel per sec.}=62{\cdot}4av\left\{h-\frac{\alpha l}{g}-\frac{v^2fl}{2gm}\right\}\text{ft. lbs.}$$

But $v=v_1+\alpha t$, so that this becomes :—

Energy entering wheel per sec.

$$=62{\cdot}4a(v_1+\alpha t)\left\{h-\frac{\alpha l}{g}-\frac{(v_1+\alpha t)^2fl}{2gm}\right\}\text{ ft. lbs.}$$

If t_1 is the length of the transition period, in seconds, so that $(v_2-v_1)\div t_1=\alpha$, we have the total energy entering the wheel from the penstock during this period given by

$$62{\cdot}4a\int_0^{t_1}\left[(v_1+\alpha t)\left\{h-\frac{\alpha l}{g}-\frac{(v_1+\alpha t)^2fl}{2gm}\right\}\right]dt\text{ ft. lbs.}$$

an expression which is readily integrated. Usually the terms involving the co-efficient of friction f, are negligibly small, in which case we get

Energy entering wheel during transition period

$$=62{\cdot}4a\int_0^{t_1}\left[(v_1+\alpha t)\left\{h-\frac{\alpha l}{g}\right\}\right]dt\text{ ft. lbs.}$$

$$=E_1\text{ ft. lbs.}$$

If the maximum value of α, consistent with good speed regulation, be known from experience of existing plants, the foregoing expression may be solved numerically; and so far as the data at the author's disposal go, these tend to show

that the acceleration should not exceed the value given by the formula

$$a = 2\cdot4\frac{h}{l} \text{ ft. per sec. per sec.}$$

Now, for satisfactory governing, the total demand during the transition period must not be greater than the supply entering at the penstock together with the energy rendered available by the slowing of the flywheel and rotating parts, so that if ω_1 and ω_2 are the angular velocities before throwing on the load, and after the speed has settled down under the new load, we have

$$E_1 + \tfrac{1}{2}I\{\omega_1^2 - \omega_2^2\} = e_2 t_1$$

where e_2 is the demand for energy, *per second*, under the increased load.

3. Effect of Stand Pipe on Speed Regulation.

Suppose a stand pipe, area A sq. ft., to be fitted at the entrance to the turbine casing, so that h_s is the head in the stand pipe.

Then with steady flow at vel. v_1 . . . $h_s = h - \frac{v_1^2}{2g}\left\{1 + \frac{fl}{m}\right\}$ ft.

and with steady flow at vel. v_2 . . . $h_s = h - \frac{v_2^2}{2g}\left\{1 + \frac{fl}{m}\right\}$ ft.

So that, during the transition period, a volume of water $\left\{1 + \frac{fl}{m}\right\}\left\{\frac{v_2^2 - v_1^2}{2g}\right\} A$ cub. ft. has flowed into the turbine casing from the stand pipe under a mean head h' feet, and has carried in an amount of energy approximately equal to

$$\left(1 + \frac{fl}{m}\right)\left\{\frac{v_2^2 - v_1^2}{2g}\right\} Ah' \text{ ft. lbs.}$$

$$= E_s \text{ ft. lbs.}$$

Here $h' = h - \frac{v_2^2 + v_1^2}{4g}\left\{1 + \frac{fl}{m}\right\}$ approx.

In this case then, we have, for satisfactory speed regulation,

$$E_1+\tfrac{1}{2}I\{\omega_1^2-\omega_2^2\}+E_s=e_2t_1$$

an expression from which I may be obtained if ω_1, ω_2, a, and A are known, or from which the necessary value of A may be obtained if the other factors are known.

As an example, consider a turbine supplied under a head of 80 feet through a penstock 4 feet in diameter and 250 feet long, and working under a normal load of 400 B.H.P. A stand pipe is to be designed to keep the speed within 4 % of the normal (250 revs.), under an increase of load up to 600 B.H.P., the turbine being fitted with a fly-wheel whose moment of inertia $\dfrac{Wr^2}{g}=7000$ foot lb. units.

Assume the efficiency to be constant, and equal to ·80. This necessitates a supply of energy under normal load . $=\dfrac{400\times 550}{\cdot 8}=275{,}000$ ft. lbs. per sec.

Also since the energy then entering the wheel casing per second . . $=62\cdot 4av_1\left\{h-\dfrac{v_1^2fl}{2gm}\right\}$

where $a=12\cdot 57$ sq. ft.; $h=80$ ft.; $f=\cdot 004$ say; this gives us on substitution and reduction :—

$$v_1=4\cdot 41 \text{ ft. per sec.}$$

while with steady flow under the increased load

$$v_2=6\cdot 64 \text{ ft. per sec.}$$

Now in the first case the head at the stand pipe $=h-\dfrac{v_1^2}{2g}\left\{1+\dfrac{fl}{m}\right\}$ ft.

$$=80-\cdot 303\{1+1\}$$
$$=79\cdot 394 \text{ ft.}$$

while in the second case the head is $h-\dfrac{v_2^2}{2g}\left\{1+\dfrac{fl}{m}\right\}$

$$=80-2\times\cdot 685$$
$$=78\cdot 630 \text{ ft.}$$

$\therefore$ Fall in level at stand pipe = ·764 ft.

The mean height in stand pipe = 79 ft., so that if A is its area, the energy leaving it during the transition

$$=\cdot 764\times 79\times 62\cdot 4\times A \text{ ft. lbs.}$$
$$=3760A \text{ ft. lbs.}$$

Now putting $\alpha=2\cdot 4\dfrac{h}{l}=2\cdot 4\times\dfrac{80}{250}=\cdot 768$ f.s.s., so that the velocity after an interval of t seconds from throwing on the load is $4\cdot 41+\cdot 768t$

ft. per sec., we get the energy entering the casing from the penstock during the transition period of t_1 seconds (E_1, p. 32), given by

$$62{\cdot}4a\int_0^{t_1}(4{\cdot}41+{\cdot}768t)\left\{80-\frac{{\cdot}768\times 250}{32}\right\}dt \text{ foot lbs.}$$

$$=784\int_0^{t_1}(327+57t)\,dt \text{ foot lbs.}$$

$$=784\{327t_1+28{\cdot}5t_1^2\} \text{ ft. lbs.}$$

On putting $t_1=\dfrac{6{\cdot}64-4{\cdot}41}{{\cdot}768}=2{\cdot}9$, this expression reduces to

$$784\{950+83\}$$

$$=800{,}000 \text{ ft. lbs.}$$

If now ω is the mean angular velocity of the runner in radians per second, and if δE is the energy given out by the wheel in slowing down through $\delta\omega$, it is easily shown that

$$\frac{\delta\omega}{\omega}=\frac{\delta E}{1\omega^2}.$$

But $\dfrac{\delta\omega}{\omega}={\cdot}04$; and $\omega=\dfrac{2\pi\times 250}{60}$;

$$\text{so that } \delta E=\frac{{\cdot}04\times 7000\times \pi^2\times 625}{9}$$

$$=192{,}000 \text{ ft. lbs.}$$

Now under the increased load, the demand per second is

$$\frac{600\times 550}{{\cdot}8}=412{,}500 \text{ ft. lbs.}$$

so that for effective regulation we have

$$3760A+800{,}000+192{,}000=412{,}500\times t_1$$
$$=1{,}196{,}000$$

$$\therefore\quad A=54{\cdot}3 \text{ sq. ft.}$$

corresponding to a diameter at the top, of 8 ft. 4 inches. This stand pipe would probably take the form of a vertical pipe 4 ft. in diameter and carrying a circular cistern at the top 8 ft. 4 in. diameter. The top of this would be about 82 ft. above the centre of the turbine, and its bottom about 77 ft. above the same level.

The method here outlined is equally applicable to the case of a Pelton Wheel or a Pressure Turbine, and while the results obtained are admittedly only rough approximations, yet they are sufficiently near for all practical purposes.

CHAPTER III

Sudden stoppage of Motion—Theory—Valve closed quickly but not instantaneously—Experimental results—Sudden closure in a non-uniform Pipe Line—Sudden opening of a Valve—Sudden opening, neglecting effect of elasticity.

1. Sudden Stoppage of Motion—Ideal Case.

If a column of water, flowing with velocity v along a uniform pipe (supposed rigid), have its motion checked by the instantaneous closure of a rigid valve, the phenomena experienced are due entirely to the elasticity of the column, and are analogous to those obtaining in the case of the longitudinal impact of an elastic bar against a rigid wall.

At the instant of impact, the motion of the layer in contact with the valve is suddenly stopped, and its kinetic energy is changed into resilience, or energy of strain, with a consequent sudden rise in pressure. This stoppage and rise in pressure is almost instantaneously transmitted to the adjacent layer, and so on, the state of zero velocity and maximum pressure (this at any point being p' above the pressure obtaining at that point with steady flow at velocity v) being propagated as a pressure-wave along the pipe, with velocity V_p. [V_p is the same as the velocity of sound through water, *i.e.* about 4700 ft. per second, depending slightly on temperature.]

This wave reaches the open end of the pipe after t seconds, where $t = l \div V_p$. At this instant the whole of the column is instantaneously at rest in a state of compression.

At the open end, however, a constant pressure p_1 is maintained, and in consequence the strain energy of the end layer is reconverted into kinetic energy, this (neglecting losses), rebounding with its original velocity v and with the normal pressure obtaining at this point under a state of steady flow towards the open end with this velocity.

This state of normal pressure and of velocity $(-v)$ is then propagated as a wave towards the valve, reaching the latter after a second interval $l \div V_p$ seconds. At this latter instant the whole of the column is unstrained and is moving towards the open end with velocity v. At the same instant the motion of the layer nearest the valve is stopped, and a wave of zero velocity and of pressure (p' below the pressure obtaining at the point at the instant before the stoppage of the motion, or p below the pressure at the point with no flow through the pipe) is transmitted along the pipe to be reflected from the open end as a wave of normal pressure and velocity v towards the valve. When this wave reaches the valve, $4l \div V_p$ seconds after the latter is closed, the conditions are the same as at the beginning of the cycle and the whole is repeated.

Under such ideal conditions the state of affairs behind the valve, as regards pressure, would be represented by such a diagram as Fig. 9 *A*, the cycle, in the case of an elastic, non-viscous fluid, being repeated indefinitely. At any other point in the pipe, at a distance l_1 from the open end, the pressure-time diagram would appear as in Fig. 9 *B*.

Actually the valve does not close instantaneously, while the stretching of the pipe and the viscosity of the water cause the maximum pressure attained to be less than in the ideal rigid pipe, and also cause the pressure waves to diminish rapidly in amplitude. The state of affairs is then

as indicated in Fig. 10, of which A represents a diagram from behind the valve, and B from a point 15 feet from the open end of the pipe line experimented on by the author. In this case the valve was closed in ·07 seconds, and the

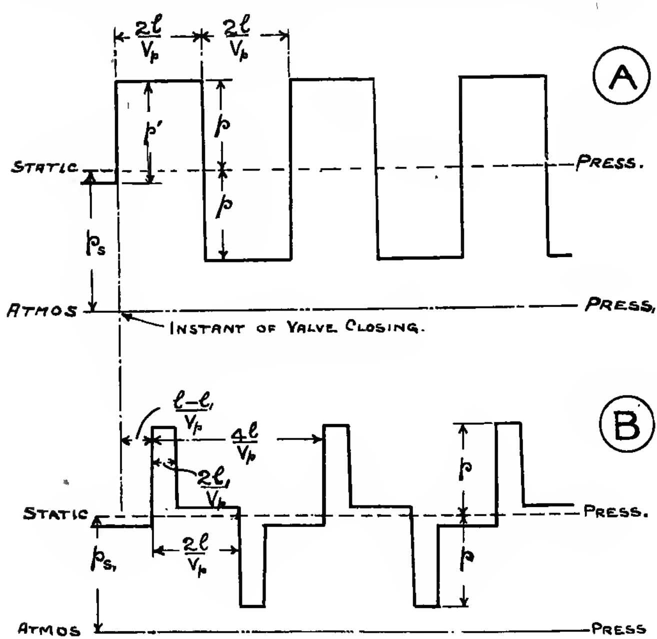

FIG. 9.

vibrations died out so that the motion of the pencil of the indicator became imperceptible, after about 30 complete oscillations.

Magnitude of rise in pressure at the sudden closing of a Valve.

If p' be the rise in pressure in lbs. per sq. ft., and if v be the velocity of flow at the instant of stoppage (supposed

instantaneous), we have, assuming the pipe line rigid, on equating the loss of kinetic energy per lb. to the increase in resilience :—

$$\frac{v^2}{2g}=\frac{p'^2}{2Kw},$$

$$\therefore \quad p'=v\sqrt{\frac{Kw}{g}} \quad \ldots\ldots\ldots\ldots \quad (19)$$

Putting $K=300{,}000\times 144$ lbs. per sq. ft.; $w=62{\cdot}4$; $g=32{\cdot}2$ this becomes $p=9160\ v$ lbs. per sq. ft.
$=63{\cdot}7\ v$ lbs. per sq. inch.

A closer approximation to the actual rise in pressure may be obtained by assuming that while the pipe line is rigid in

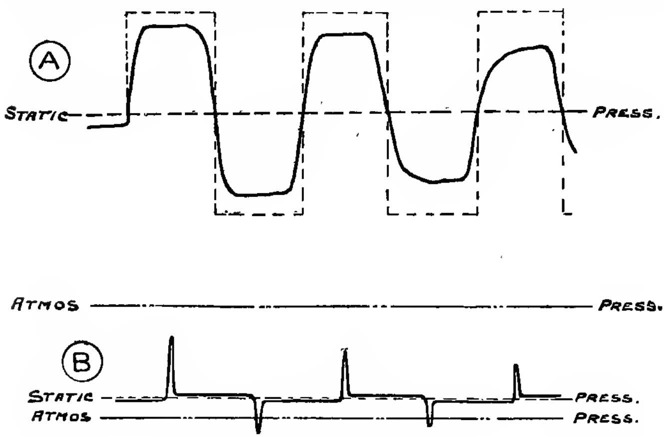

FIG. 10.

that the motion is instantaneously stopped, yet the elasticity is felt in its effect on the value of K, this adopting the value K' where $\frac{1}{K'}=\frac{1}{K}+\frac{r}{2tE}\left(5-\frac{4}{\sigma}\right)$. P. 15.

In the experimental pipe line this makes $K'=251{,}000\times 144$ lbs. per sq. ft., and makes $p'=58{\cdot}4\ v$ lbs. per sq. inch.

The following demonstration shows how the elasticity of the pipe line and water column may be taken fully into account.

Let K' and E have the meanings already attached to them, and let w and w_m, V and V_m, a and a_m, be the weights of unit volume of, the velocities of wave propagation in, and the sectional areas of the water column and metal of the pipe wall respectively.

Then, with instantaneous closure the ends of the water and metal columns move, at impact, with a common velocity u, and waves, respectively of compression and of extension, travel along the water column and the pipe wall.

Hence, after a very short interval of time δt, lengths $V\delta t$ and $V_m\delta t$ of the water column and of the pipe will be moving with velocity u, and the equation of momentum gives us:—

$$\{waV + w_m a_m V_m\}u\delta t = waVv\delta t$$

$$\therefore \quad u = v\left\{\frac{1}{1+\dfrac{w_m a_m V_m}{waV}}\right\}$$

Each element of the column and of the pipe, as the wave passes it, takes suddenly the velocity u, while each element of the water column takes the compression $\dfrac{v-u}{V}$ and therefore the stress $(v-u)\sqrt{\dfrac{wK'}{g}}$, and each element of the pipe takes the extension $\dfrac{u}{V_m}$ and the stress $u\sqrt{\dfrac{w_m E}{g}}$.

Substituting for u we have the pressure rise in the water given by

$$p' = v\left\{\frac{1}{1+\dfrac{waV}{w_m a_m V_m}}\right\}\sqrt{\frac{wK'}{g}} \text{ lbs. per sq. ft.}$$

Since $V=\sqrt{\dfrac{K'g}{w}}$ and $V_m=\sqrt{\dfrac{Eg}{w_m}}$ * this may be written

* Imagine a bar of unit cross-sectional area to impinge with velocity v in the direction of its axis, against a rigid wall. After

$$p' = v\left\{\frac{1}{1+\frac{a}{a_m}\sqrt{\frac{K'w}{Ew_m}}}\right\}\sqrt{\frac{wK'}{g}}$$

$$= v\left\{\frac{1}{\sqrt{\frac{g}{K'w}}+\frac{a}{a_m}\sqrt{\frac{g}{Ew_m}}}\right\} \text{lbs. per sq. ft.} \quad \ldots\ldots (20)$$

The longitudinal stress f, produced in the pipe walls by hammer action, which equals $u\sqrt{\frac{w_m E}{g}}$, then becomes, on substitution,

$$f = v\left\{\frac{1}{\sqrt{\frac{g}{Ew_m}}+\frac{a_m}{a}\sqrt{\frac{g}{K'w}}}\right\} \text{lbs. per sq. ft.}$$

while since the circumferential stress in a pipe exposed to pressure is twice the longitudinal stress, the maximum increment of stress in the metal, due to this action, is equal to 2 f.

2. Valve shut suddenly, but not instantaneously.

As the time of closure of a valve becomes less and less, the maximum rise in pressure will evidently tend to the limit given by formula 20, p. 41.

Now if the time of closure is so short that $\frac{l}{V_p} > T = \frac{x}{V_p}$,

---

a very short interval δt seconds, a mass $\frac{w_m V_m \delta t}{g}$ has been brought to rest, and, if p is the (uniform) pressure on the end of the bar during this interval we have, equating the force × time, to the change of momentum:—

$$p\delta t = \frac{w_m V_m \delta t}{g} \cdot v \text{ or } p = \frac{w_m}{g} \cdot V_m \cdot v \text{ lbs.}$$

But $p = v\sqrt{\frac{Ew_m}{g}}$, so that, equating these two expressions we get $V_m = \sqrt{\frac{Eg}{w_m}}$ ft. per second.

the disturbance initiated at the valve has travelled a distance x, and has not arrived at the open end when the latter reaches its seat.

In this case if the retardation is uniform $(=-a)$, equation (17), p. 17, becomes $p'=\frac{w}{g}\int_0^x a\,dx=\frac{wax}{g}$ lbs.

But $a=\frac{v_a}{T}=\frac{v_a V_p}{x}$, so that

$$p'=\frac{w}{g}\cdot V_p v_a=v_a\sqrt{\frac{K'w}{g}}\text{ lbs.,}$$

this being the value obtaining with instantaneous stoppage. It follows that whatever the law of valve closing, if this is completed in a less time than $l\div V_p$ the pressure rise will be the same as with instantaneous closure.

For values of T between $l\div V_p$ and $2l\div V_p$ the falling off in pressure will usually be comparatively small, so that it is in general sufficiently accurate for all practical purposes to count as 'sudden,' any stoppage occupying a shorter time than this.

For values of $T>2l\div V_p$, formula (1), p. 3, when modified for the effect of elasticity as indicated on p. 17, is to be used, while this elasticity correction may reasonably be neglected where $T>4l\div V_p$.

3. Experimental Results with sudden closing of the Valve.

Taking $K'=251{,}000\times 144$; $E=10{,}000{,}000\times 144$ lbs. per sq. ft.; $\frac{a}{a_m}=1{\cdot}275$; (p. 16), in formula (20), this becomes

$$p=7780\,v\text{ lbs. per sq. foot}$$
$$=54{\cdot}0\,v\text{ lbs. per sq. inch.}$$

For comparison the results observed by the author are shown in Table I. against those obtained by using this formula.

TABLE I.

Experiment	Velocity before closing Valve.	Time of Closing.	Rise in Pressure. Calculated ($p=54v.$)	Rise in Pressure. Experimental.
	ft. per sec.	secs.	lbs. per sq. in.	lbs. per sq. in.
1	·3625	·070	19·5	19·5
2	,,	·090	,,	19·5
3	,,	·095	,,	19·5
4	,,	·140	,,	19·5
5	,,	·140	,,	18·7
6	,,	·270	,,	16·3
7	,,	·280	,,	14·7
8	·551	·065	29·7	29·3
9	,,	·090	,,	29·3
10	,,	·145	,,	29·3
11	,,	·175	,,	29·3
12	,,	·255	,,	25·0
13	,,	·275	,,	24·4
14	,,	·275	,,	26·2
15	·720	·125	38·9	37·5
16	,,	·135	,,	38·1
17	,,	·150	,,	38·1
18	,,	·250	,,	37·0
19	,,	·270	,,	35·7
20	,,	·270	,,	35·7
21	1·094	·110	59·0	57·5
22	,,	·150	,,	58·7
23	,,	·245	,,	55·0
24	,,	·285	,,	46·3
25	1·444	·160	77·9	75·0
26	,,	·165	,,	75·0
27	,,	·195	,,	71·5
28	,,	·210	,,	73·7
29	,,	·215	,,	73·0
30	,,	·250	,,	63·7
31	,,	·300	,,	62·5
32	,,	·370	,,	58·8

From these results it appears that so long as T is less than ·13 seconds ($l \div V_p = \cdot 13$) the calculated and observed pressures are in every case in close agreement. This agreement is substantially maintained until T=about ·21 seconds, while when $T = \cdot 26$ seconds, the mean error involved in using the uncorrected formula in this case is about 14%.

Other Experimental Results.

In a series of experiments carried out by M. Joukowsky * on cast-iron pipes of 4″ and 6″ diameter, having lengths of 1050 and 1066 feet respectively, the time of valve closing being ·03 seconds in each case, the observed rise in pressure agrees closely with the formula, $p = 57v$. The following are some of the results obtained by interpolation from the plotted results of these experiments.

4-INCH PIPE.

Vel.—ft. per sec.	·5	2·0	3·0	4·0	9·0
Observed pressure—lbs. per sq. in. .	31	119	172	228	511
$p=57v$	29	114	171	228	513

6-INCH PIPE.

Vel. ft. per sec.	·6	2·0	3·0	7·5
Observed pressure—lbs. per sq. in. .	43	113	173	426
$p=57v$	34	114	171	427

* *Stoss in Wasserleitungsröhren*, St. Petersburg, 1900. An abstract of this paper by O. Simin is given in *The Trans. Am. Waterworks Ass.*, 1904.

4. Sudden Closure of a Valve in a Pipe Line of non-uniform section.

In such a case the phenomena become very complicated. Let l_1, l_2, l_3, etc., be the lengths of successive sections of the pipe, of areas a_1, a_2, a_3, and suppose the valve to be at the extremity of the length l_1. Imagine the pipes rigid and the water incompressible. Following sudden closure of the valve, a wave of zero velocity and of pressure ($63{\cdot}7v_1$ lbs. per sq. inch above normal) is then transmitted to the junction of pipes 1 and 2. Here the pressure changes suddenly to $63{\cdot}7v_2$ above normal. This is maintained during the passage of the wave through the second pipe, and is followed by a change of pressure to $63{\cdot}7v_3$ at the junction of 2 and 3, and so on to the end of the pipe line. But immediately the pressure at the junction of 1 and 2 attains its value $63{\cdot}7v_2$, the wave in pipe 1 is reflected back to the valve, as a wave of pressure $63{\cdot}7v_2$ and of velocity $v_1 - v_2$, being again reflected from the valve as a wave of zero velocity and pressure $63{\cdot}7\{v_2 - (v_1 - v_2)\}$ above normal.

This wave then travels to and fro along the pipe 1, making a complete journey in $l_1 \div V_p$ seconds, until such time as the wave in pipe 2, reflected from the junction of 2 and 3 with pressure $63{\cdot}7v_3$ above normal and with velocity $v_2 - v_3$, again reaches the junction of 1 and 2. This occurs at an instant $l_2 \div V_p$ seconds after its reflection. At this instant it takes up a velocity and pressure corresponding to the velocity and pressure at the junction end of pipe 1, and as this pressure and velocity may be either positive or negative depending on the ratio of the lengths of the branches 1 and 2, it is evident that after the first passage of the wave the pressure conditions at any particular instant are practically indeterminate. The greater the number of varia-

tions in area and the more involved does the phenomenon become.

Where a pipe is very short, the period of the oscillations of pressure at any point becomes so small that the pencil of the ordinary indicator is unable to record them, and simply records the mean pressure in the pipe.

Thus where a short branch of comparatively small diameter is used as the outlet from a long pipe of larger bore, the pressure as recorded by an indicator will be sensibly the same at any instant, as in the large pipe at the point of attachment of the outlet branch.

This point, as well as the effect of an enlargement of the pipe section on the magnitude of the hammer pressure, is well brought out by the following results of experiments by E. B. Weston.*

In each experiment of this series the outlet valve was closed suddenly, but the exact time of closing is not given.

Series I. Pipe line consisting of $\begin{cases} 111 \text{ ft. of } 6'' \text{ pipe.} \\ 58 \quad ,, \quad 2'' \quad ,, \\ 99 \quad ,, \quad 1\frac{1}{2}'' \quad ,, \\ 4 \quad ,, \quad 1'' \quad ,, \end{cases}$

The outlet valve is on the 1″ length.

Pressures have been calculated from the formula $p=60v$ lbs. per sq. inch.

Vel. in 1″ pipe —ft. per. sec.	Pressures—lbs. per sq. inch.					
	1″ pipe.		$1\frac{1}{2}$″ pipe.		6″ pipe.	
	Calc.	Obs.	Calc.	Obs.	Calc.	Obs.
2·57	154	73	69	71	...	...
5·36	322	129	143	127	8·9	14·5
10·05	...	...	...	...	16·8	23·5
19·23	...	...	...	...	32·0	51·7

* American Society of Civil Engineers, Nov. 19, 1884.

Series II. Pipe line consisting of

111	ft. of	6″	pipe.
58	„	2″	„
48	„	1½″	„
3	„	3″	„
48	„	1½″	„
4	„	1″	„

The outlet valve is on the 1″ length.

Vel. in 1½″ pipe —ft. per sec.	Pressures—lbs. per sq. inch.					
	1st 1½″ pipe.		3″ pipe.		2nd 1½″ pipe.	
	Calc.	Obs.	Calc.	Obs.	Calc.	Obs.
1·19	71·5	75	18·0	65	71·5	61
2·37	142	126	35·5	121	143	114
3·00	180	150	45	150	180	139
4·47	268	203	67	207	268	196

Series III. Pipe line consisting of

182	ft. of	6″	pipe.
66	„	4″	„
4	„	2½″	„
1	„	2″	„
7	„	1½″	„
6	„	1″	„

The outlet valve is on the 1″ length.

Vel. in 1″ pipe —ft. per sec.	Pressures—lbs. per sq. inch.							
	1″ pipe.		1½″ pipe.		2½″ pipe.		6″ pipe.	
	Calc.	Obs.	Calc.	Obs.	Calc.	Obs.	Calc.	Obs.
5·39	323	67	120	49	52	22	9·0	4·8
6·71	403	76	149	62	64·5	36	11·2	6·6
10·02	601	106	223	82	97	52	16·7	15·8
20·94	1255	177	466	122	201	99	35·0	36·8
43·9	...	...	...	...	421	183	73·2	80·1

5. Sudden opening of a Valve.

If the valve at the lower end of a pipe line be suddenly opened, the pressure behind the valve falls by an amount p lbs. per sq. inch, and a wave of velocity v towards the valve $\left\{v=p\sqrt{\frac{g}{K'w}} \text{ (approx.)}\right\}$, and of pressure p below statical, is propagated towards the pipe inlet.

The magnitude of p depends on the speed and amount of opening of the valve, and if the latter could be thrown wide open instantaneously the pressure would fall to that obtaining on the discharge side. In the author's experiments, with the valve thrown open through ·5 of a complete turn the maximum drop in pressure was 40 lbs. per sq. inch, the statical pressure being 45 lbs. per sq. inch.

With the valve opened through ·10 of a complete turn the maximum drop was 20 lbs. per sq. inch, and with $\frac{1}{20}$ of a complete turn the drop was 11 lbs. per sq. inch. In each case the time of opening was less than ·13 seconds ($l \div V_p$).

In the case of a horizontal pipe, or one which is so situated that the absolute statical pressure is everywhere greater than p, this pressure wave reaches the pipe inlet with approximately its original amplitude, and at this instant the whole column is moving towards the valve with velocity v and pressure p below normal.

The pressure at the inlet is, however, maintained normal, so that the wave returns from this end with normal pressure and with velocity $2v$. At the valve this wave is reflected with a velocity which is the difference between $2v$ and the velocity of efflux at this instant, and with a corresponding pressure. As the velocity of efflux will now be greater than v, the wave velocity will be less than v, and the rise in pressure less than p above normal. This wave is reflected

from the inlet to the valve, and here the cycle is repeated, the amplitude of the pressure wave diminishing rapidly until steady flow ensues.

Fig. 11 shows a diagram obtained by the author from the experimental pipe line under these conditions.

Where the pipe slopes upwards towards its inlet, so that, beyond a certain point in its length the absolute statical pressure is less than the drop in pressure caused at the valve by sudden opening, then on the passage of the first wave of

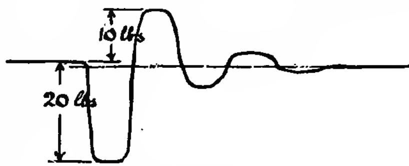

FIG. 11.

negative pressure the wave motion becomes partially discontinuous after this point is reached,* and the wave travels on

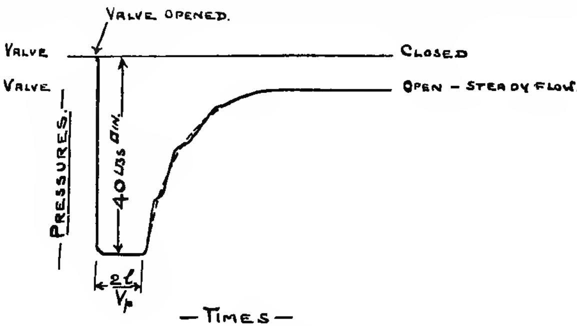

FIG. 12.

to the inlet with a gradually diminishing amplitude. The amplitude with which it reaches the inlet, and which will be probably 2 or 3 lbs. per sq. in. less than the absolute statical pressure at inlet, decides the state of velocity of the reflected

* Actually before this, since water gives up its dissolved air rapidly when the pressure falls to within 2 or 3 lbs. of a complete vacuum.

wave. This will evidently be less than in the preceding case, and under such circumstances the wave motion dies out very quickly. As the valve opening becomes greater, the efficiency of the valve as a reflecting surface becomes less, so that with a moderate opening the pressure may never even attain the pressure due to the statical head.

This is shown in Fig. 12, which is a diagram obtained by the author from the experimental pipe line when the outlet valve was opened suddenly (time $<\cdot 13$ secs.) through half a complete turn.

6. Sudden Opening neglecting Elasticity of Water Column.

Neglecting the effect of elasticity of the water column, the pressure behind the valve and the velocity of efflux accompanying a sudden valve opening may be calculated as follows:

Imagine the effective valve opening to instantaneously assume a value a_0 and to remain of this value, so that $\frac{dv_a}{dt}=\frac{a_0}{a}\cdot\frac{dv_0}{dt}$. Equation (6), (p. 9), now becomes

$$\frac{p}{w}+\frac{v^2}{2g}+z=-\frac{1}{g}\int_0^x\frac{dv}{dt}\,dx-\frac{f}{2gm}\int_0^x v^2dx+C \quad . \; . \; . \; . \; . \quad (6'')$$

while equation (9) becomes

$$-a_0\frac{dv_0}{dt}=\frac{a}{l}\left\{\frac{v_0^{\,2}}{2}\left(1+\frac{fl}{m}\left(\frac{a_0}{a}\right)^2\right)-gh\right\} \quad . \; . \; . \; . \; . \; . \quad (9'')$$

or $k\,\dfrac{dv_0}{dt}=c^2-v_0^{\,2}$

$$\text{where}\quad\begin{cases} k=\dfrac{2la_0}{a}\cdot\dfrac{1}{1+\dfrac{fla_0^{\,2}}{ma^2}} \\[2ex] c^2=\dfrac{2gh}{1+\dfrac{fl}{m}\cdot\dfrac{a_0^{\,2}}{a^2}} \end{cases}$$

so that $\int \frac{dv_0}{c^2 - v_0^2} + L = \frac{1}{k}\int dt$

From this we get

$$\frac{1}{2c}\log\frac{c+v_0}{c-v_0} + D = \frac{t}{k}, \quad \ldots \ldots \ldots \ldots \quad (11'')$$

while since $v_0 = 0$ when $t = 0$, we have $D = 0$.

$$\therefore \quad v_0 = c\left\{\frac{1 - e^{-\frac{2c}{k}\cdot t}}{1 + e^{-\frac{2c}{k}\cdot t}}\right\}\text{ft. per sec.}, \quad \ldots \ldots \ldots \quad (12'')$$

giving the velocity of efflux at an instant t seconds after the valve opens.

As t increases, this approximates to the value c or

$$\sqrt{\frac{2gh}{1 + \frac{fl}{m}\cdot\frac{a_0^2}{a^2}}}.$$

By substitution in (9″) the value of $\frac{dv_0}{dt}$ at any instant and therefore of $\frac{dv}{dt}$ may be obtained, and knowing this the pressure behind the valve may be readily obtained.

A series of experiments recently carried out at the University of Wisconsin* on a hydraulic ram, in which the velocity of flow down the drive-pipe, corresponding to a given interval of valve opening was measured, enable the results obtained by using formula (12″), or its modification,

$$v_a = C\frac{a_0}{a}\left\{\frac{1 - e^{-\frac{2c}{k}\cdot t}}{1 + e^{-\frac{2c}{k}\cdot t}}\right\}\text{ ft. per sec.},$$

to be verified.

Values of v_a as obtained by experiment on a drive-pipe 85·35 ft. long and 2·05 inches diameter, are plotted in Fig. 13 against the curve representing this formula. In this case $f = \cdot 09$ (by experiment); $a \div a_0 = 3\cdot 97$.

* *Bulletin of the University of Wisconsin*, No. 205, 1908, p. 143.

The valve used consisted of a flat disc 4·5 ins. in diameter, the inside diameter of its seat being 2·75 ins. Its lift was

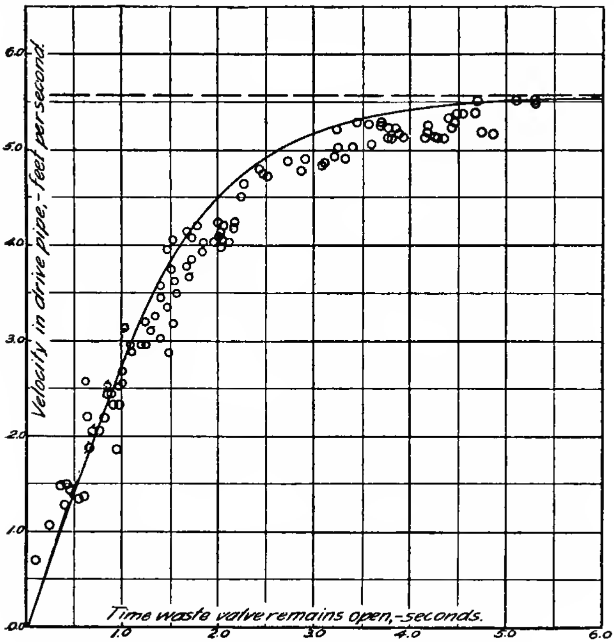

FIG. 13.

·5 ins., and in every case the time of opening was too short to be measured.

The experimenter states that in these experiments the recorded velocities are somewhat low owing to the effect of the friction of the rod carrying the disc used to measure the velocity.

The agreement between observed and calculated values is, considering the nature of the apparatus, very close indeed.

CHAPTER IV

1. FURTHER PHENOMENA CONNECTED WITH PIPE FLOW.

WHILE carrying out the experiments already described, further phenomena were observed by the author which would appear to be of some importance when considering the question of the fluctuations of pressure likely to be obtained in a given pipe line.

To follow these more clearly it should be understood that the water after passing the main regulating valve *A* (Fig. 14), passes through a second valve *B*, before being measured. In the experiments already described the valve *B* was kept wide open. This valve differs from *A* in only one particular.

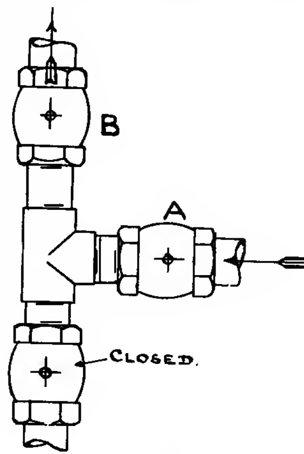

FIG. 14.

Instead of bedding metal to metal on its seat, a thin leather washer, $\frac{1}{16}''$ thick, is fixed to the valve body as shown in Fig. 15, and is thus interposed between the valve and its seat.

In one or two experiments, the valve *A* being wide open, *B* was opened slightly. Whenever this was done it was noticed that pressure waves were set up and were reflected from end to end of the pipe, but it was found that these,

instead of dying out actually increased in amplitude up to a certain maximum, after which this amplitude was maintained seemingly indefinitely.

It having been noted within what range of opening of *B* this occurred, *A* was closed and *B* set to some point within this range. *A* was then opened very slowly so as not to set up any perceptible disturbance of the indicator lever. Flow would then take place steadily for a few seconds or even minutes, but always, after a short time the pencil would tremble slightly, and the oscillations would grow until the amplitude approached that obtained by a sudden opening of the valve (as much as ± 28 lbs. per sq. inch).* In fact experiment showed that over a certain range of valve opening it was impossible to maintain steady flow.

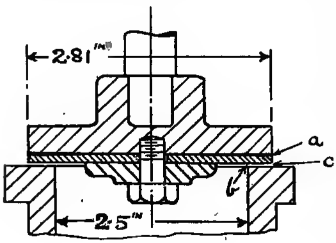

FIG. 15.

On entirely closing valve *B*, the oscillations died out, however, in about 30 seconds. In every case the period of a complete oscillation of the pencil lever was ·26 seconds.

When it is remembered that the pressure of the water tends to keep the valve off its seat, this phenomenon seems, at first sight, very paradoxical.

Its explanation may, however, be found in the fact of the leather washer fixed to the valve body, for while the pressure in the annular space between this washer and the valve body is the same as that on the outlet side of the valve—at *a* or *c*, Fig. 15—that between the washer and the valve seat will be

* Data obtained from a number of the experiments are given in the Appendix, Table II.

less than this (by Bernoulli's theorem), since the velocity of flow at c is less than at any point between c and b. Also, the greater the volume passing the valve per second at any instant, the greater will be the pressure at a and at c, and, for a given valve opening, the greater will be the difference of pressure between the two sides of the washer, tending to force the latter to its seat.

Now a sudden opening is followed by a reduction of pressure behind the valve and flow is initiated. When this attains a certain value v, depending on the valve opening, the difference of pressure on the two sides of the washer becomes sufficient to cause it to flutter to its seat. The pressure rises suddenly due to the stoppage of motion and a wave of pressure p above normal is reflected to the open end of the pipe. Here it is reflected with normal pressure and velocity $-v$. This is reflected from the valve with pressure p below normal and zero velocity, and again from the open end with normal pressure and velocity v towards the valve.

But following the drop in pressure, the washer will leave its seat and flow is set up, so that this state of velocity v which is propagated towards the valve is superposed on a velocity of flow v'. As the valve flutters to its seat under the influence of the increasing pressure, the maximum pressure rise will now be proportional to $v+v'$ instead of to v.

The cycle as outlined above is thus repeated, but with a pressure variation which becomes greater at each succeeding cycle until such time as a balance is obtained between the energy entering the pipe during each cycle and that expended in stretching its walls and in giving kinetic energy to the escaping water.

The initiation of this phenomenon from a state of steady flow is in all probability due to pressure fluctuations caused

by uneven eddy formation on the discharge side of the valve.

The above explanation is borne out by the fact that it was found impossible to obtain these oscillations with valve *A*, or with valve *B* when the leather washer was removed.

Evidently this state of affairs may exist in any main fitted with an outlet valve of a type similar to that illustrated, and it would appear very inadvisable to use any such valve on a branch pipe communicating with a pipe line used for supplying a prime mover, or in any other case where (as when coupled up to a water meter) such a fluctuation of pressure, accidentally set up, would prove detrimental.

APPENDIX

TABLE I.

Series A. Valve open ·62 of a complete turn ; $\frac{a}{a_1}=19{\cdot}5$; $\frac{flv_a^2}{2gm}=12{\cdot}3$ lbs. per sq. in. ; $\bar{v}_0=3{\cdot}03$ ft. per sec. ; $h=104{\cdot}6$ ft.

Expt.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Calculated rise in pressure above that obtaining with steady flow at velocity $\bar{v}_0$ in lbs. per sq. in. . .	90	70	58	56	48	42	35	33	31	26·5	22·5	18	17	17	16·2
Observed rise in pressure, lbs. per sq. in. . .	84	64	54	57·5	47·5	40	37	33	33	26	22·5	18·5	16·7	17·5	15·5
Time of closure in seconds	·67	·80	·98	1·02	1·25	1·38	1·75	1·92	2·05	2·5	3·4	5·7	7·2	7·4	8·6

Series B. Valve open ·349 of a complete turn ; $\frac{a}{a_1}=34{\cdot}7$; $\frac{flv_a^2}{2gm}=6{\cdot}2$ lbs. per sq. in. ; $\bar{v}_0=2{\cdot}155$ ft. per sec. ; $h=104{\cdot}6$ ft.

Expt.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Calculated rise in pressure, lbs. per sq. in. . .	91	80	63	39	25	24	21	19	13·5	12·5	10·0	8·7	8·6	7·9	7·5
Observed rise in pressure, lbs. per sq. in. . .	85	80	61	·40	25	24	22	18	13·5	12·0	10·2	8·7	8·2	7·7	7·2
Time of closure in seconds,	·35	·40	·48	·70	1·10	1·20	1·40	1·60	2·7	3·1	4·7	6·6	7·1	10·7	13·0

TABLE I.—*continued.*

Series C. Valve open ·167 of a complete turn ; $\frac{a}{a_1}=72{\cdot}5$; $\frac{flv_a^2}{2gm}=1{\cdot}6$ lbs. per sq. in. ; $\bar{v}_0=1{\cdot}095$ ft. per sec. ; $h=105{\cdot}5$ ft.

Expt.	1	2	3	4	5	6	7
Calculated rise in pressure, lbs. per sq. in.,	62	55	37	22	19	9·0	5·5
Observed rise in pressure,	62	55	39	23	19	9·5	6·0
Time of closure, in seconds,	·220	·245	·35	·50	·60	1·30	2·40

Series D. Valve open 1·41 complete turns ; $\frac{a}{a_1}=8{\cdot}6$; $\frac{flv_a^2}{2gm}=7{\cdot}8$ lbs. per sq. in. ; $\bar{v}_0=2{\cdot}405$ ft. per sec. ; $h=104$ ft.

Expt.	1	2	3	4	5	6	7	8	9
Calculated rise in pressure, lbs. per sq. in.,	52	36	30·5	28·5	25	23	17	15·5	14·5
Observed rise in pressure,	45	35·5	29	29	25	23	17·2	15·7	14·5
Time of closure in seconds,	2·0	3·0	3·85	4·2	5·0	5·6	8·0	9·0	10·0

TABLE II.

Experimental Results on water-ram caused by sudden or gradual opening of valve B.

Experiment.		Quantity passing valve in lbs. per min. when flow has become steady.	Maximum oscillation of pressure—lbs. per sq. in.	Remarks.
Valve opened suddenly	1.	3·5	±22·5	Oscillations die out in 75 seconds.
	2.	6·8	±21·0	Amplitude maintained for 1 hour, valve then closed.
	3.	8·8	±30·0	Amplitude fell to ± 24 lbs. in 1 hour, afterwards maintained constant for 3 hours.
	4.	18·0	±19	Amplitude of ± 15 lbs. at end of 8 hours.
	5.	20·3	±28 8	
	6.	26·7	±20	
	7.	37		Maximum fall in pressure 28 lbs. per sq. in., no oscillation set up.
	8.	600		Max. pressure drop 40 lbs. per sq. in., no oscillations set up.
	9.	16·7	±22·5	When oscillations were in full swing, discharge fell to 13·0 lbs. per sq. in.
Valve opened slowly. Oscillations grew from zero.	10.		±12	Valve opening ·0265 of a turn.
	11.		±24·5	,, ·053 ,, ,,
	12.		±29	,, ·0795 ,, ,,
	13.		±17	,, ·106 ,, ,,
	14.		± 2	,, ·132 ,, ,,
	15.	14·1	±23	
	16.	17·4	±23	
	17.	20·7	±25	
	18.	26·0	±21	

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Printed by T. and A. Constable, Printers to His Majesty
at the Edinburgh University Press